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Model analysis methods. Composition of clans

Dmitry Zaitsev

<http://daze.ho.ua>

Modeling by Petri and Sleptsov nets

- Verification of protocols by Petri nets
- **Model analysis methods. Composition of clans**
- Analysis of Computational Grids and Clouds by Infinite Petri Nets
- Evaluation of System Performance by Colored Petri Nets
- Computing on Sleptsov networks

Application domains

- **Verification of networking protocols as basis of cybersecurity**
- **Correctness of concurrent programs**
- **Automated manufacture design**
- **Design of business processes**
- **Programmable controllers**
- **Traffic control: railway, automobile etc**

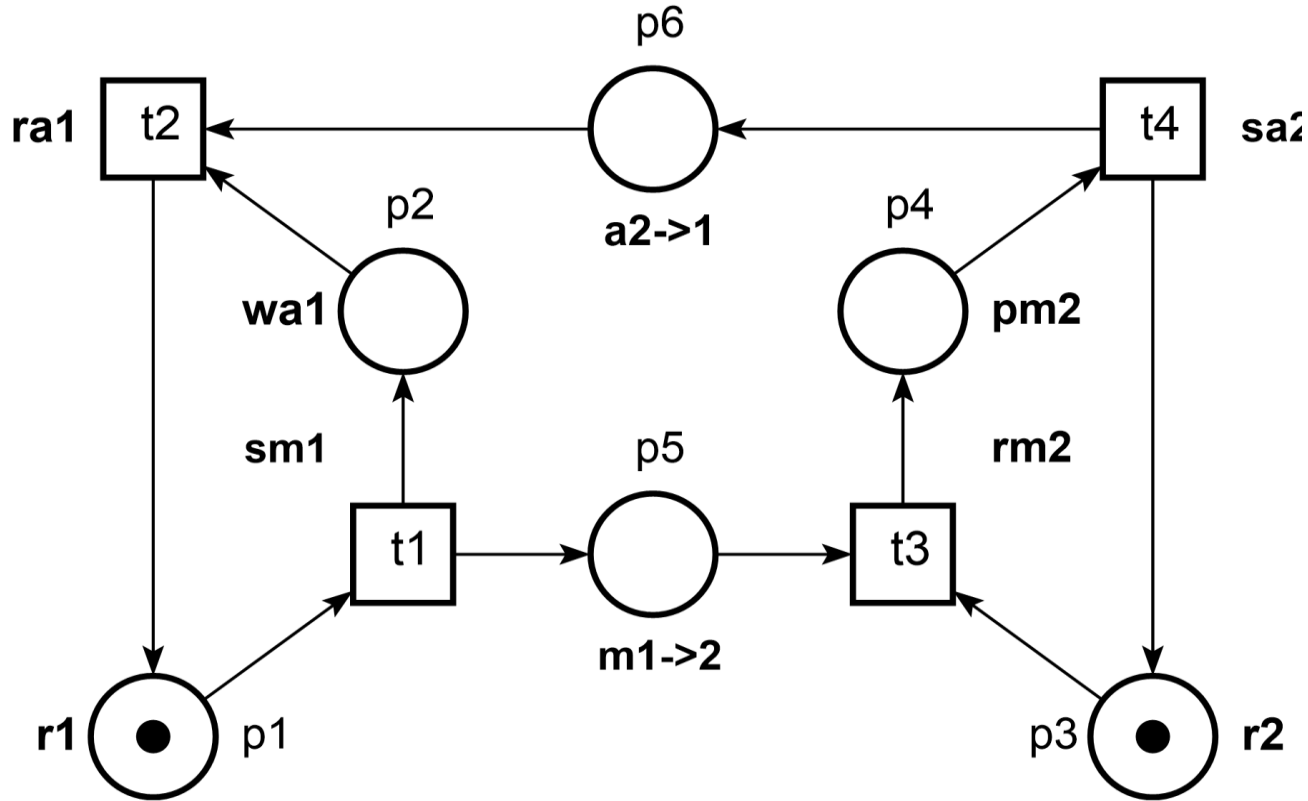
Model properties

- **Boundedness** – finite number of states
- **Safeness** – marking does not exceed a unit
- **Conservativeness** – preserves weighted sum of tokens
- **Reachability** of a given marking
- **Liveness** – from each state a marking is reachable in which a given transition is firable

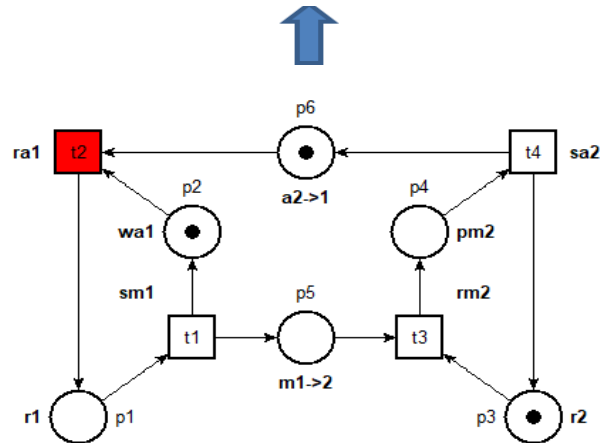
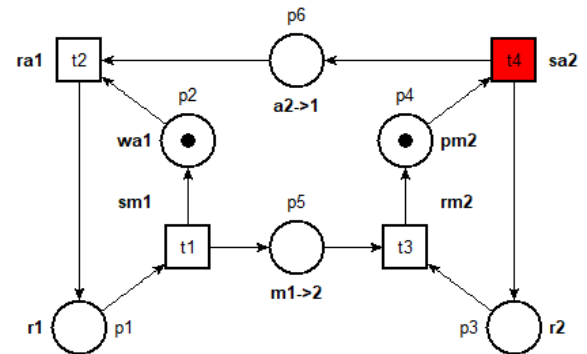
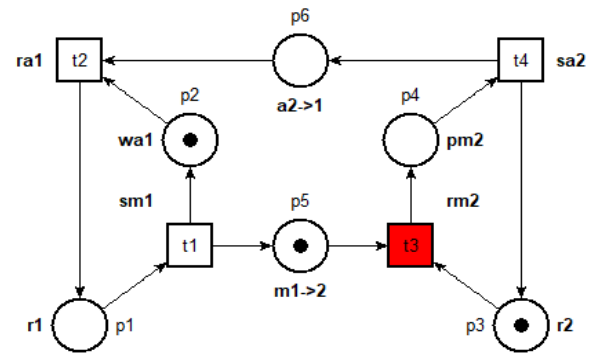
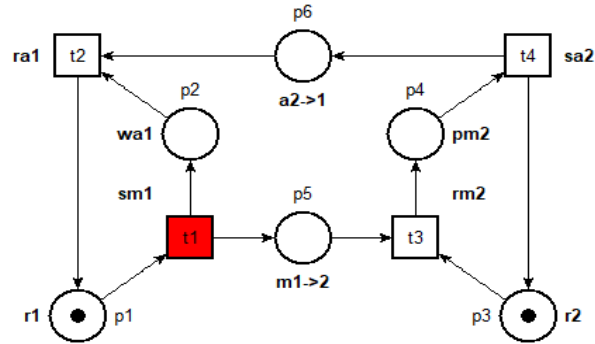
Methods of Petri net analysis

- Graphs (trees) of reachable and coverable markings
- Fundamental equation of net and linear invariants – solving linear systems in nonegative integer numbers
- Siphons and traps
- Reduction – transformations reducing the net size and preserving its properties
- Decomposition – splitting into parts

Model of protocol SimAck



Behavior of net



Model state space – marking graph

REACHABILITY ANALYSIS -----

bounded

4 marking(s), 4 transition(s)

MARKINGS:

0 : p1 p3

1 : p2 p3 p5

2 : p2 p4

3 : p2 p3 p6

REACHABILITY GRAPH:

0 -> t1/1

1 -> t3/2

2 -> t4/3

3 -> t2/0

0.000s

LIVENESS ANALYSIS -----

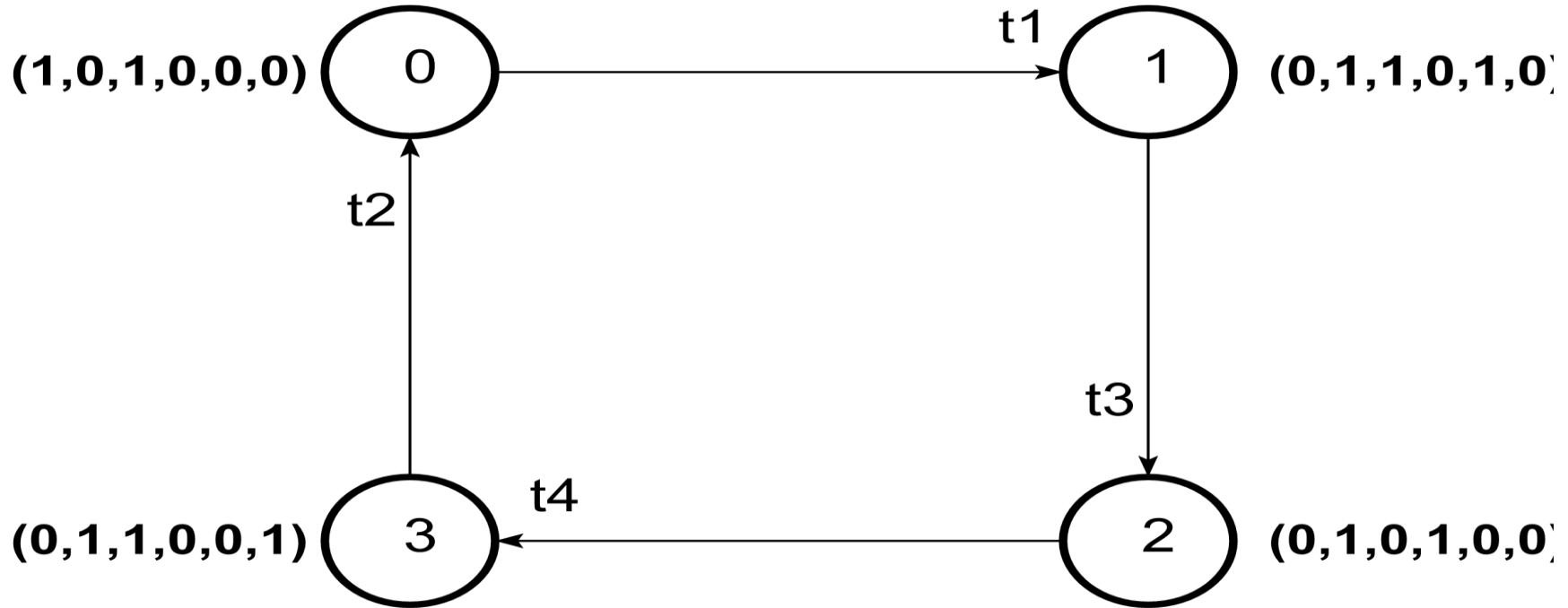
live

reversible

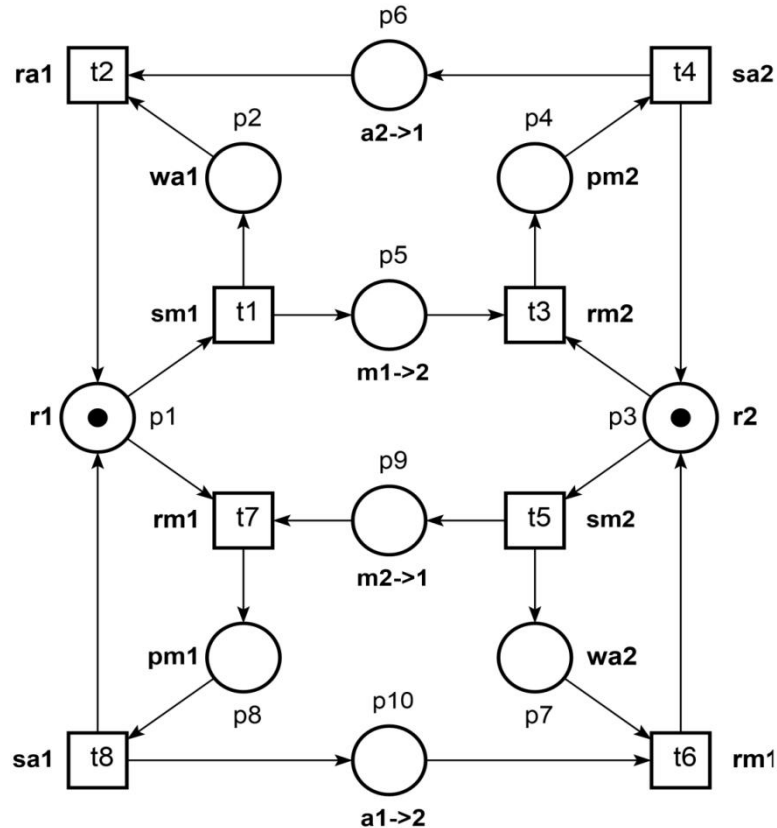
0 dead marking(s), 4 live marking(s)

0 dead transition(s), 4 live transition(s)

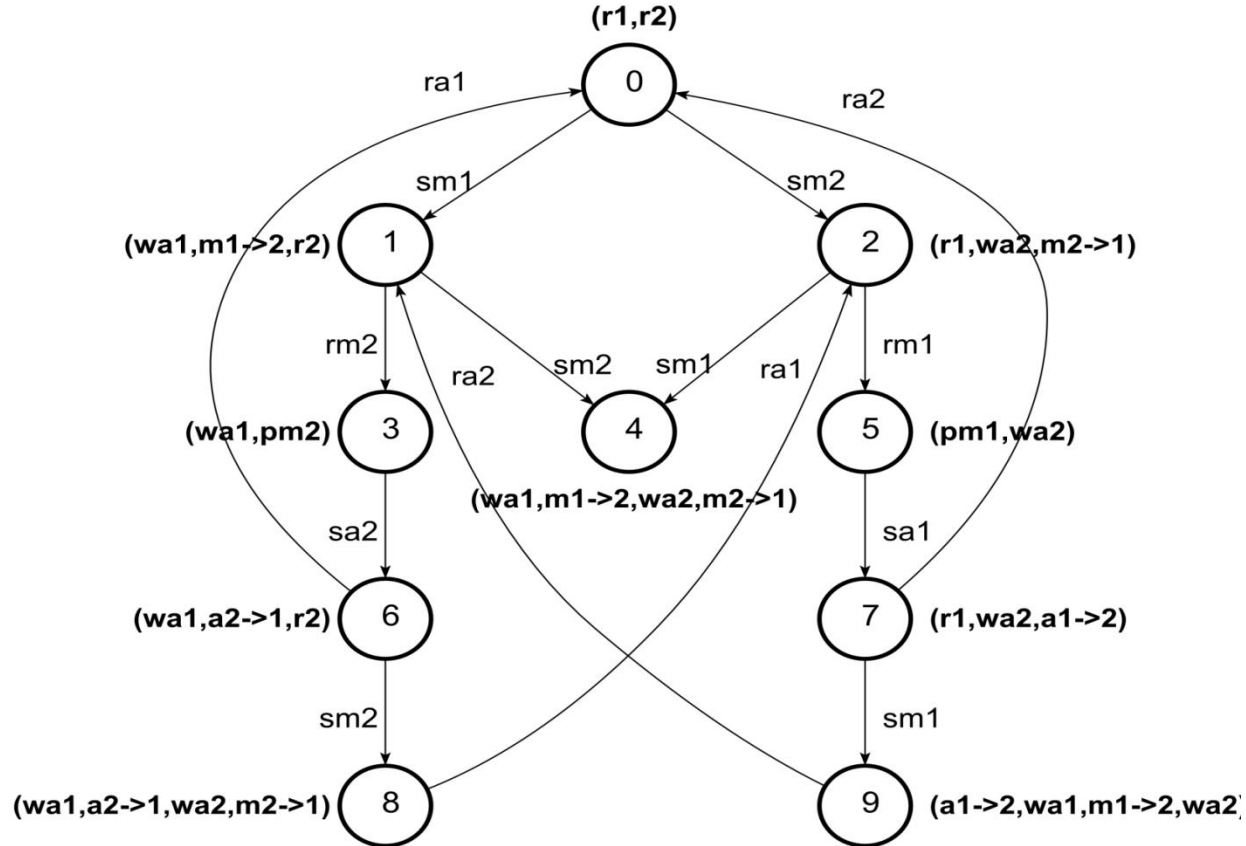
GRM: markings as vectors



Protocol of bidirectional transmission of messages with acknowledgements (DupAck)



Graph of reachable markings DupAck



Fundamental equation and linear invariants

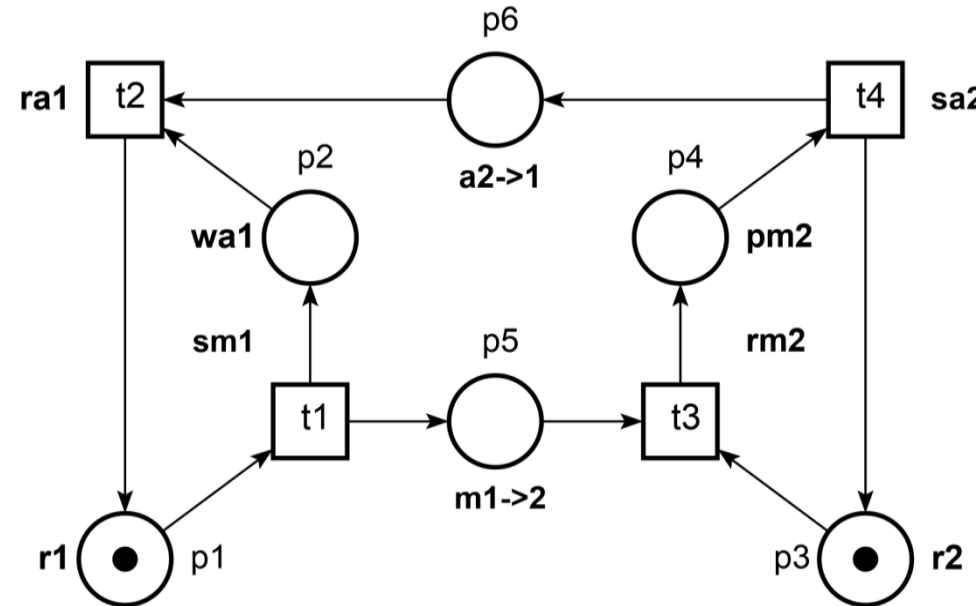
$$C\vec{z} = \overrightarrow{\Delta\mu} \quad \text{fundamental equation}$$

$$\vec{x}C = 0 \quad \text{invariants of places}$$

$$C\vec{y} = 0 \quad \text{invariants of transitions}$$

Solve in nonnegative integer numbers!

Examples of systems for computing invariants



Invariants of places

$$\begin{cases} -x_1 + x_2 + x_5 = 0 \\ -x_2 - x_6 + x_1 = 0 \\ -x_2 - x_5 + x_4 = 0 \\ -x_4 + x_2 + x_6 = 0 \end{cases}$$

Invariants of transitions

$$\begin{cases} -y_1 + y_2 = 0 \\ y_1 - y_2 = 0 \\ -y_3 + y_4 = 0 \\ y_3 - y_4 = 0 \\ y_1 - y_3 = 0 \\ -y_2 + y_4 = 0 \end{cases}$$

Basis invariants

P-SEMI-FLOWS GENERATING SET -----

invariant

p1 p2 (1)

p1 p4 p5 p6 (1)

p3 p4 (1)

0.000s

T-SEMI-FLOWS GENERATING SET -----

consistent

t1 t2 t3 t4

0.000s

ANALYSIS COMPLETED -----

Composition of clans to speed-up solving linear systems

- **Nearness relation over equations – two equations are near if they contain a variable with coefficients of the same sign**
- **Clan relation – transitive closure of nearness relation**
- **Clan relation – equivalence relation**
- **Clan – element of partitioning system via clan relation**

Decomposition of system matrix into clans

$$A = \begin{vmatrix} A^{0,1} & \widehat{A}^1 & 0 & 0 & 0 \\ A^{0,2} & 0 & \widehat{A}^2 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A^{0,k} & 0 & 0 & 0 & \widehat{A}^k \end{vmatrix}$$

The Innovative Computing Laboratory


<http://icl.utk.edu>

- Jack Dongarra – legend of supercomputer technology
- Packet LAPACK (LINPACK) – the most cited source of computer science; solving (big) linear systems; benchmarks for computer performance
- Packets for modern parallel and distributed architecture using OpenMP, MPI, CUDA

The most powerful computers in the world Top 500

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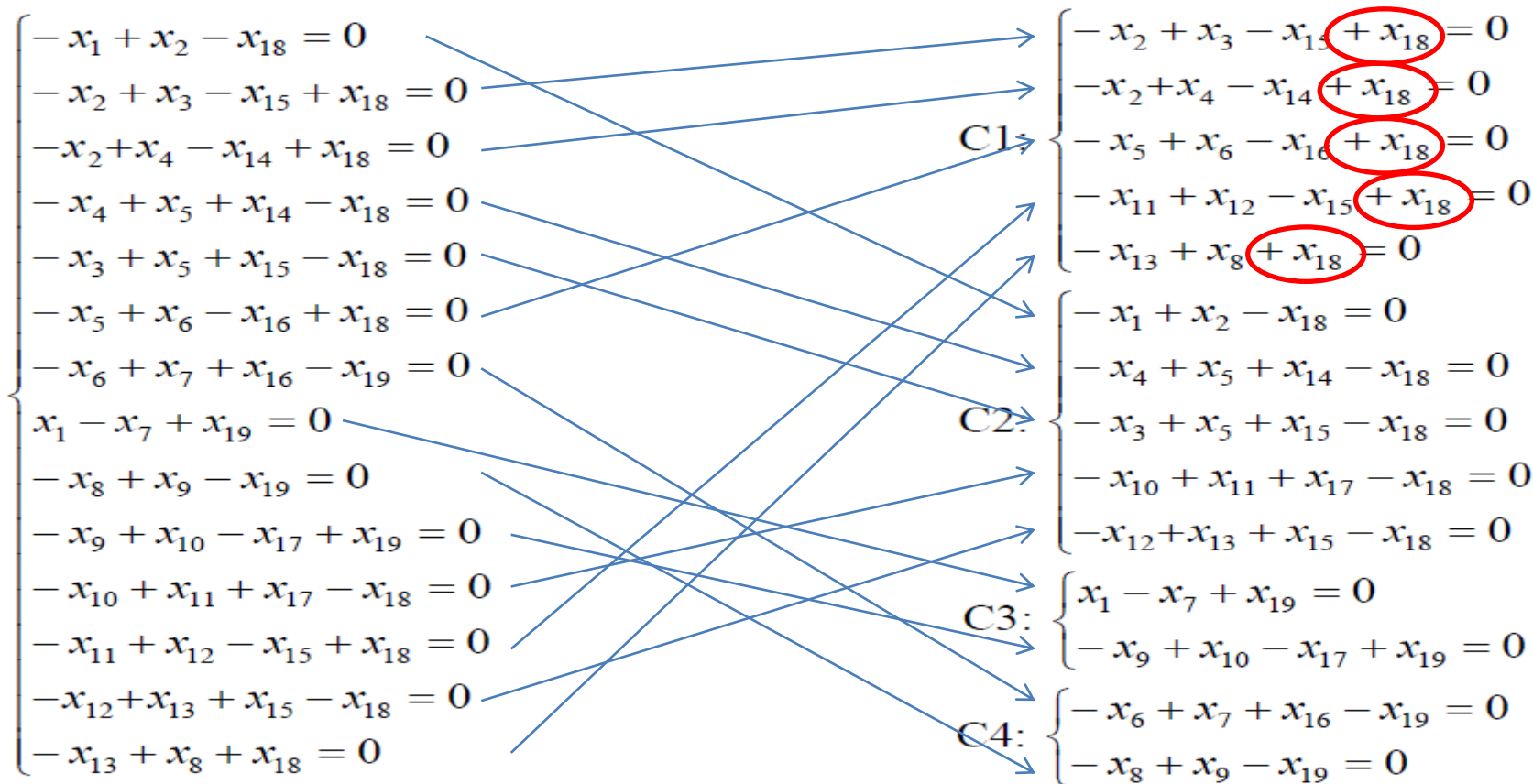
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Clan – transitive closure of nearness relation

$$C1: \begin{cases} -x_2 + x_3 - x_{15} + \underline{x_{18}} = 0 \\ -x_2 + x_4 - x_{14} + \underline{x_{18}} = 0 \\ -x_5 + x_6 - x_{16} + \underline{x_{18}} = 0 \\ -x_{11} + x_{12} - x_{15} + \underline{x_{18}} = 0 \\ -x_{13} + x_8 + \underline{x_{18}} = 0 \end{cases}$$

two equations are *near* if they contain a variable with coefficients of the same sign

Example of decomposition into clans



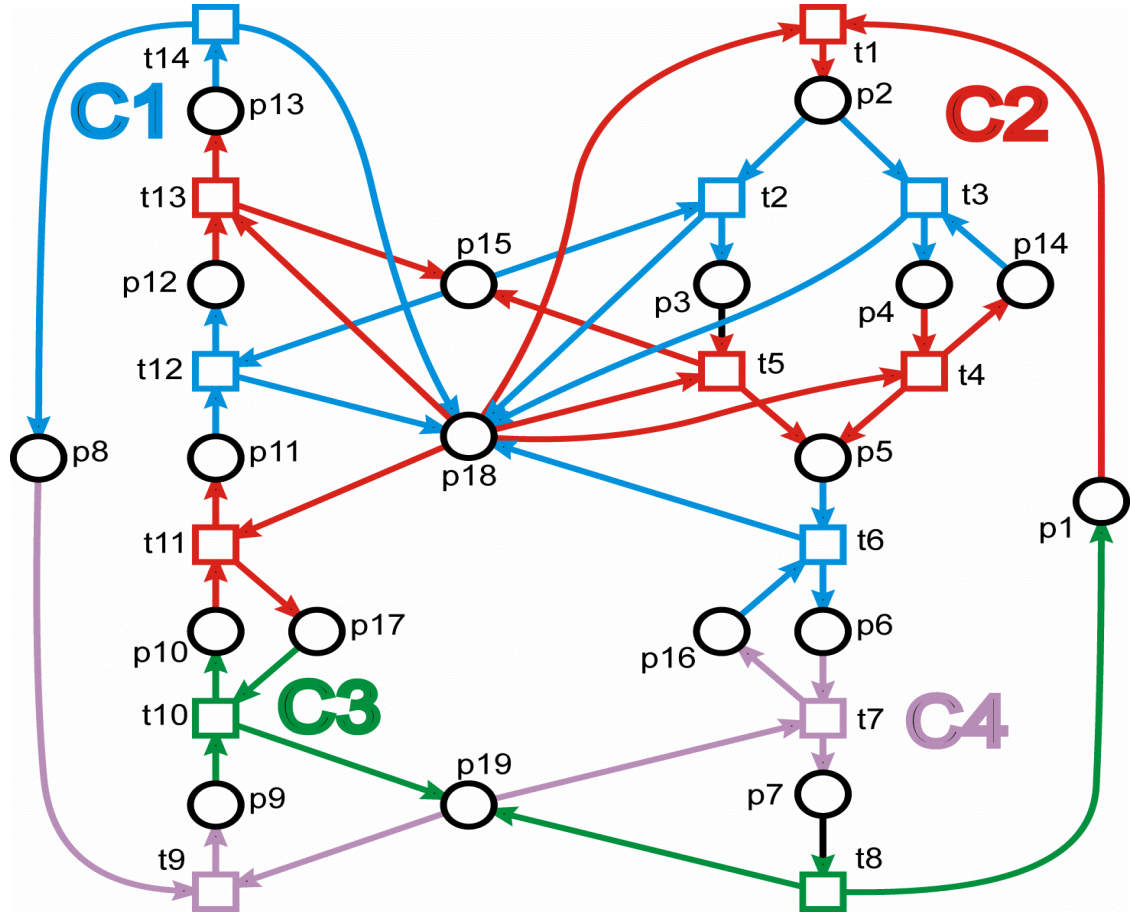
Systems and bipartite directed graphs

Equation –
transition (square)

Variable –
place (circle)

Positive sign –
incoming arc of place

Negative sign –
outgoing arc of place



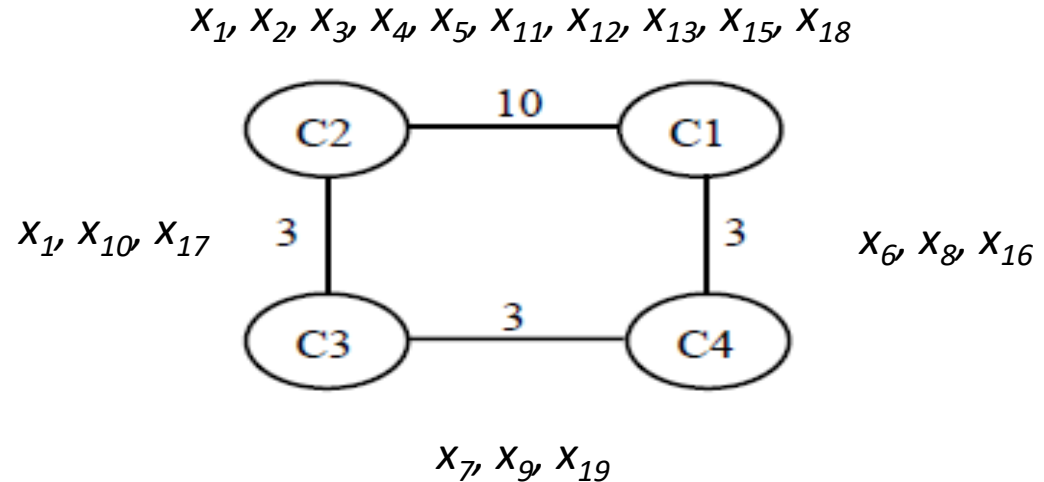
Graph of decomposition into clans

$$C1: \begin{cases} -x_2 + x_3 - x_{15} + x_{18} = 0 \\ -x_2 + x_4 - x_{14} + x_{18} = 0 \\ -x_5 + x_6 - x_{16} + x_{18} = 0 \\ -x_{11} + x_{12} - x_{15} + x_{18} = 0 \\ -x_{13} + x_8 + x_{18} = 0 \end{cases}$$

$$C2: \begin{cases} -x_1 + x_2 - x_{18} = 0 \\ -x_4 + x_5 + x_{14} - x_{18} = 0 \\ -x_3 + x_5 + x_{15} - x_{18} = 0 \\ -x_{10} + x_{11} + x_{17} - x_{18} = 0 \\ -x_{12} + x_{13} + x_{15} - x_{18} = 0 \end{cases}$$

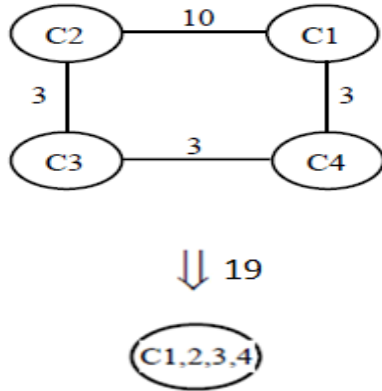
$$C3: \begin{cases} x_1 - x_7 + x_{19} = 0 \\ -x_9 + x_{10} - x_{17} + x_{19} = 0 \end{cases}$$

$$C4: \begin{cases} -x_6 + x_7 + x_{16} - x_{19} = 0 \\ -x_8 + x_9 - x_{19} = 0 \end{cases}$$

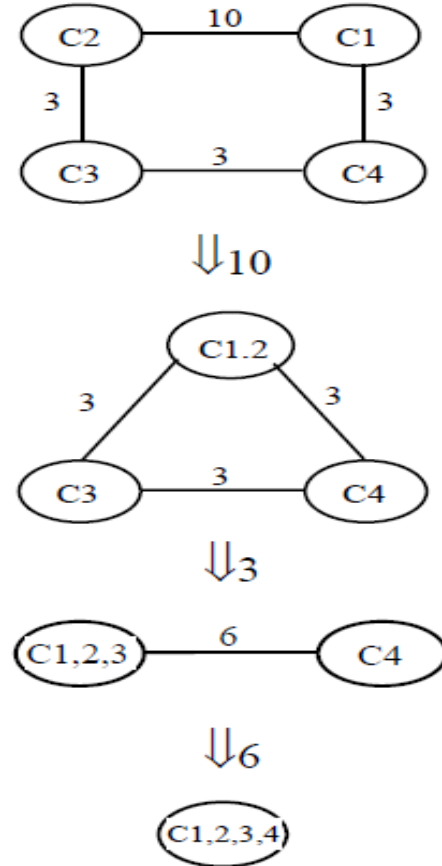


Collapse of decomposition graph

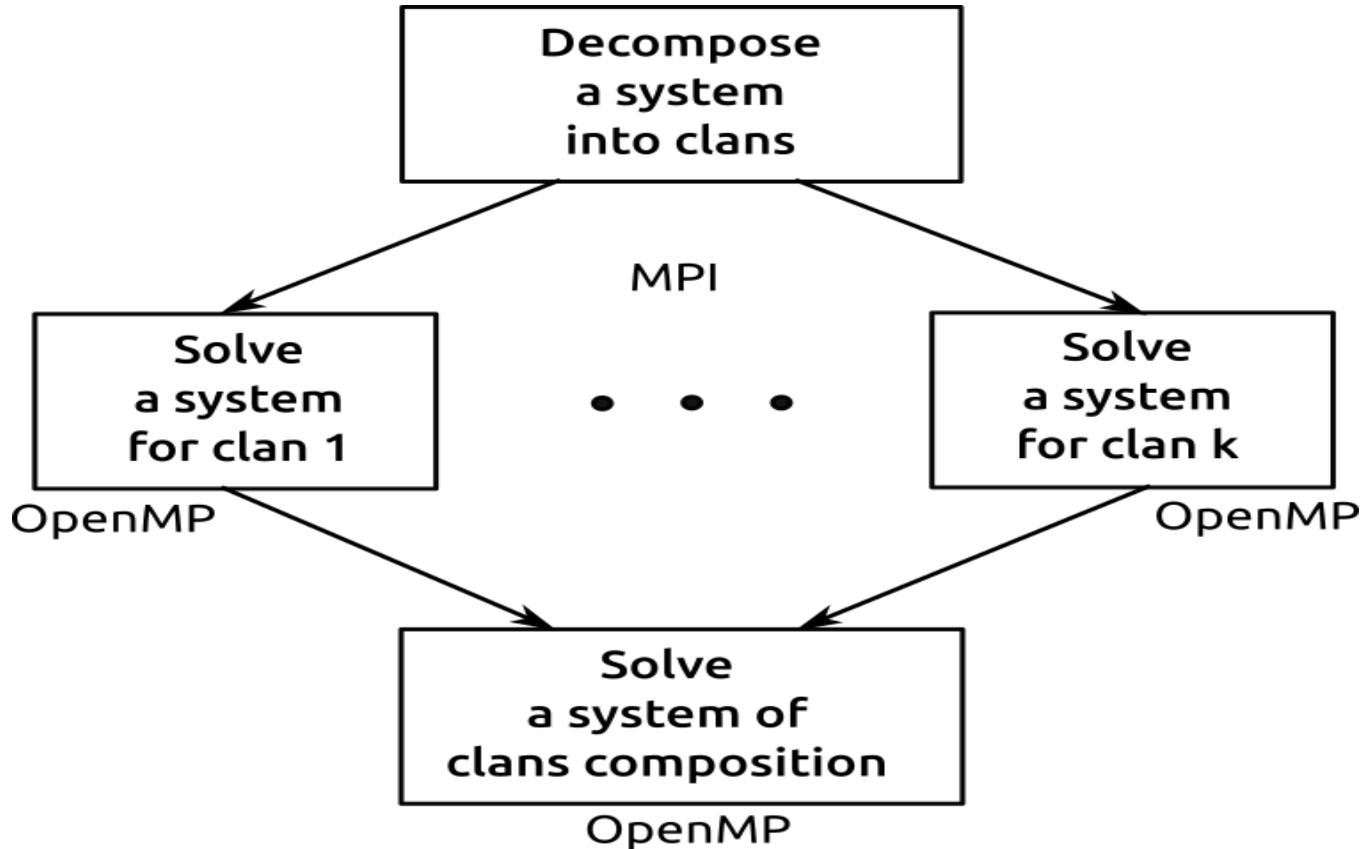
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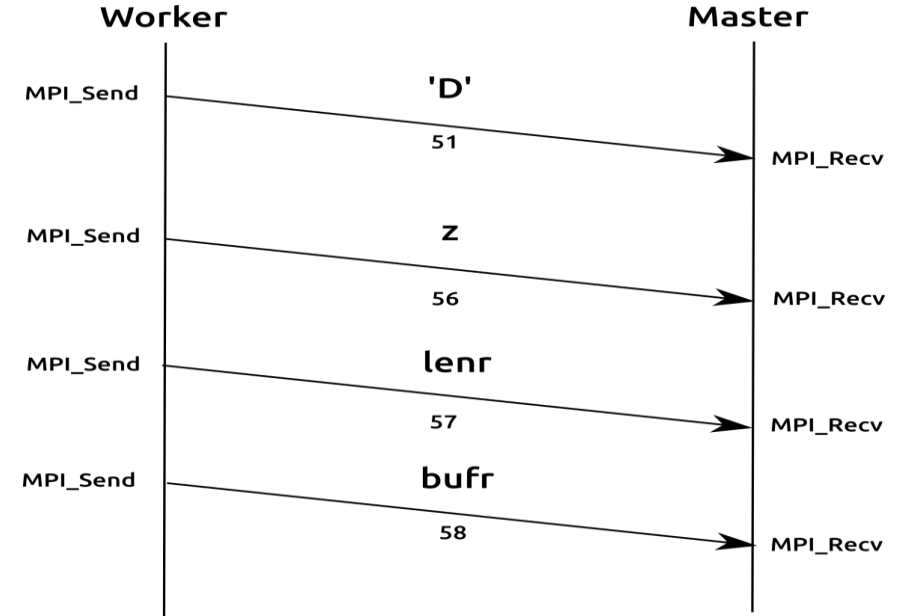
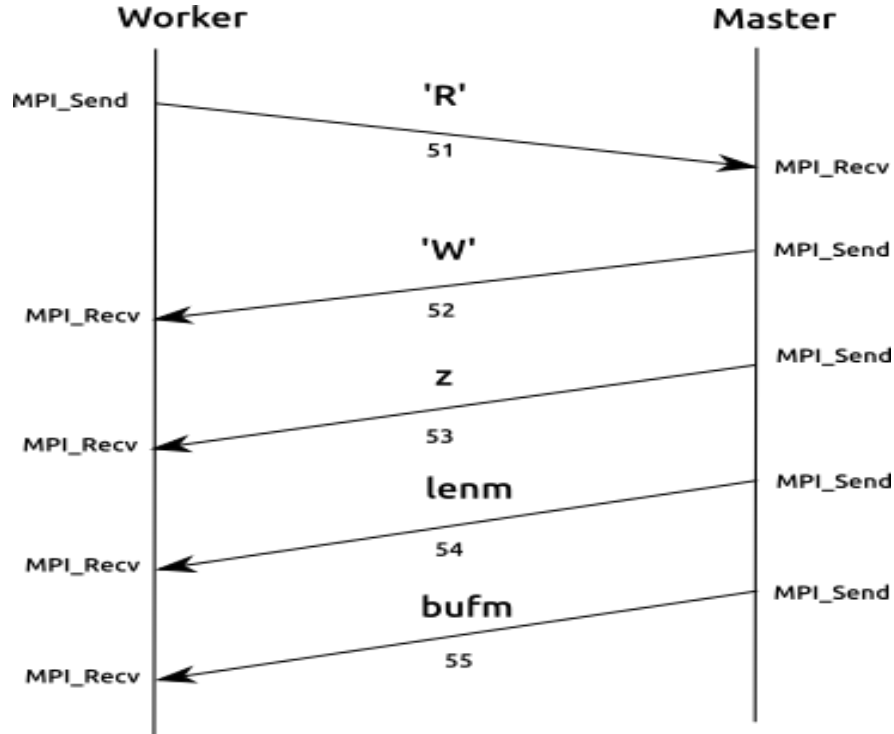
II.



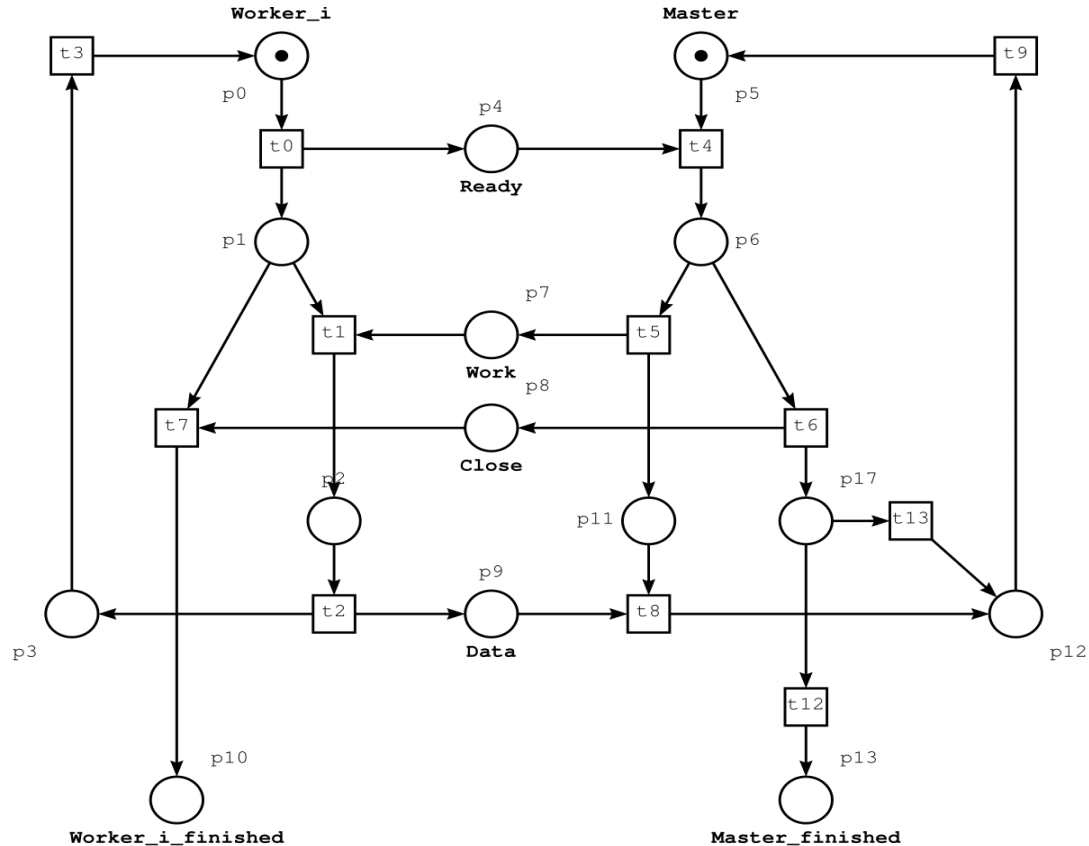
ParAd: Divide and sway



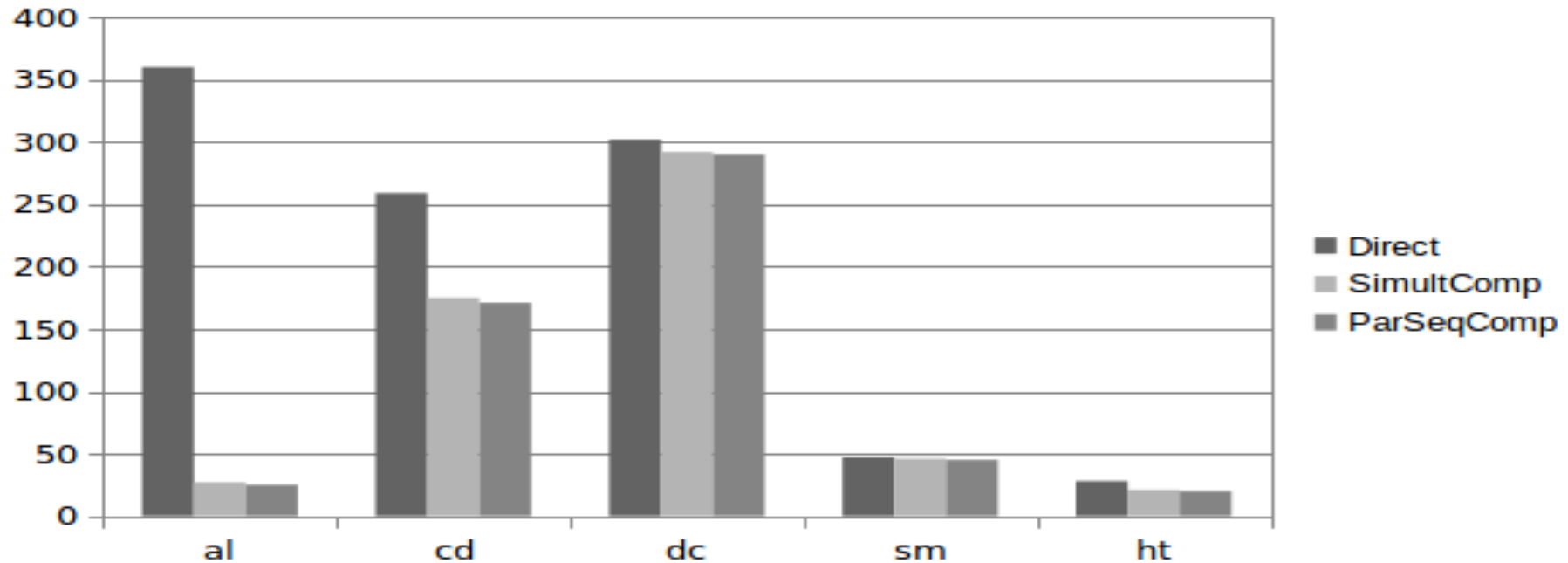
Protocols of data transmission MPI



Model of protocol



Speed-up of computations



Developed software

- **Deborah** – decomposition into clans, 2005
- **Adriana** – solving homogenous systems in process of composition (a) simultaneous or (b) sequential, 2006
- Implemented as plug-ins for Tina
<http://www.laas.fr/tina>
- **ParAd** (Parallel Adriana), 2018
<http://github.com/dazeorgacm/ParAd>

Problems

- **Development of application domains: communication systems of supercomputers – multidimensional torus, networks on chip, numerical solving partial differential equations, particle accelerators, modeling thermonuclear reaction**
- **Analysis of properties of models with a given structure and arbitrary size in multidimensional space**

References

- Dmitry A. Zaitsev, Tatiana R. Shmeleva & Piotr Luszczek, Aggregation of clans to speed-up solving linear systems on parallel architectures, International Journal of Parallel, Emergent and Distributed Systems, 37(2), 2022.
- Dmitry Zaitsev, Stanimire Tomov, Jack Dongarra. Solving Linear Diophantine Systems on Parallel Architectures, IEEE Transactions on Parallel and Distributed Systems, 30(5), 2019, 1158-1169
- Zaitsev D.A. Sequential composition of linear systems' clans, Information Sciences, Vol. 363, 2016, 292-307