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Analysis of Computational Grids and Clouds by Infinite Petri Nets

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Modeling by Petri and Sleptsov nets

- Verification of protocols by Petri nets
- Model analysis methods. Composition of clans
- **Analysis of Computational Grids and Clouds by Infinite Petri Nets**
- Evaluation of System Performance by Colored Petri Nets
- Computing on Sleptsov nets

Modern challenges

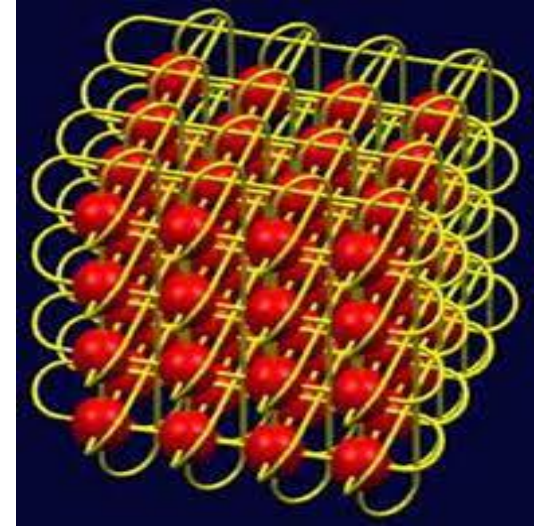
- **Connection of unlimited number of devices (systems)**
- **Spatial structure: 2D – triangle, square, hexagonal; multidimensional – hypercube and hypertorus**
- **Application area: computing grids and clouds, cellular networks of mobile communications (hexagon), television and radio broadcasting (triangle)**

Novel application domains

- **Communication subsystem of supercomputers and clusters**
- **Networks on chip**
- **Fast numerical methods: finite difference and finite elements**
- **Modeling in physics and engineering – Tokamak, particle accelerator (collider)**

Communication subsystem of supercomputers

IBM Blue Gene

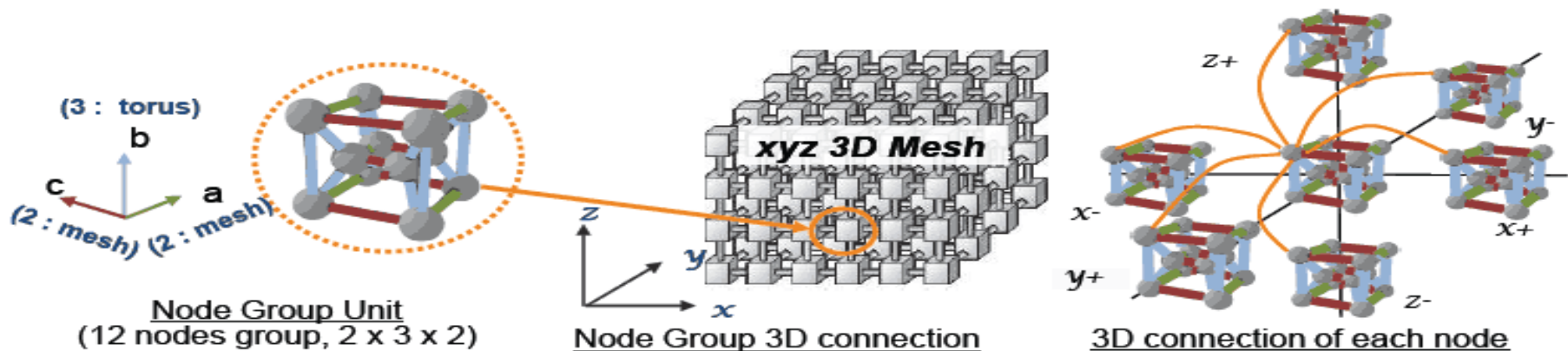


Blue Gene system with 65,000 nodes, are interconnected as a 64 x 32 x 32 three-dimensional torus. IBM Blue Gene/Q: 5D torus.

Fugaku (Fujitsu, RIKEN)



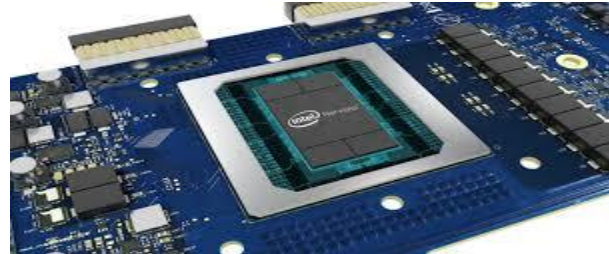
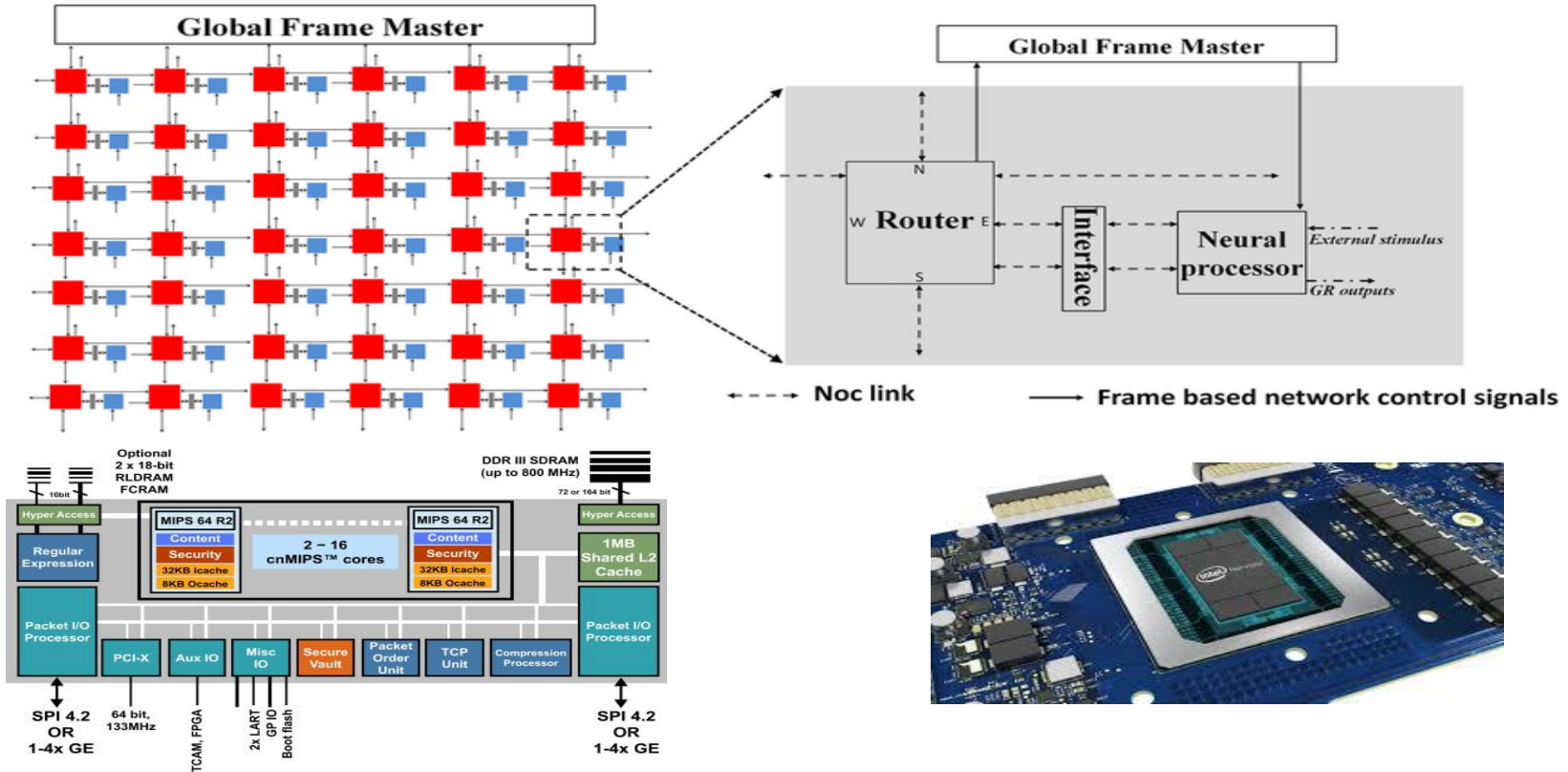
Tofu: Fujitsu's original 6D mesh/torus interconnect



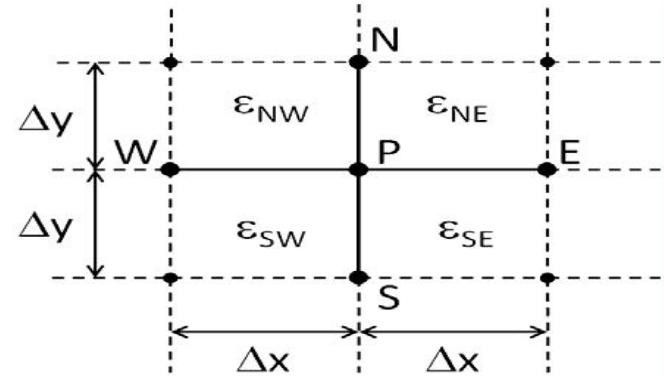
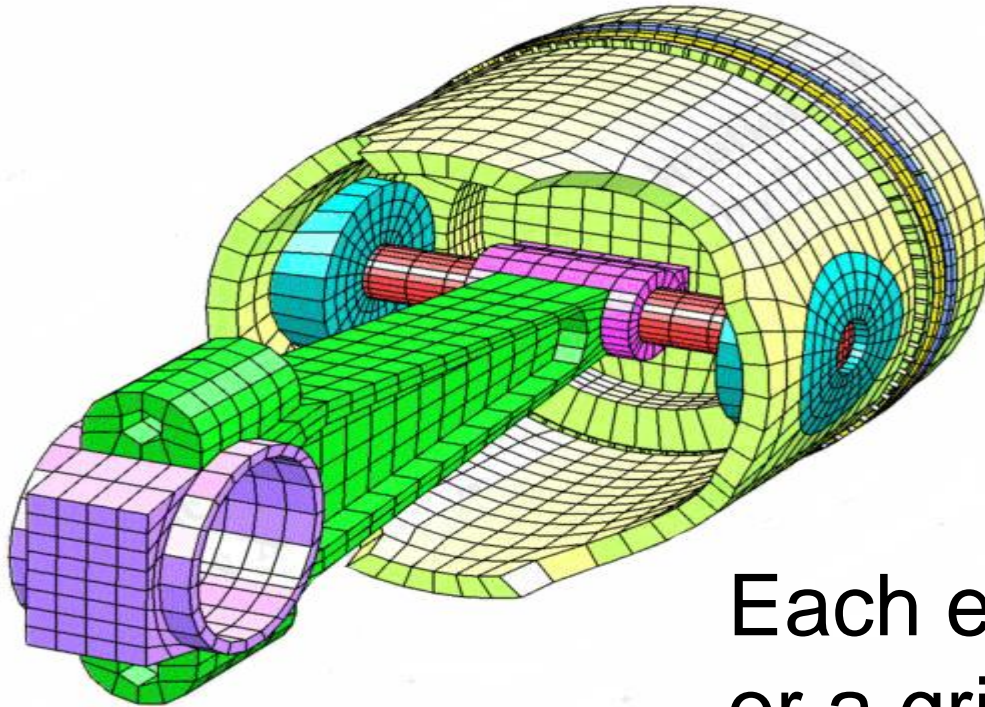
Tofu has a six-dimensional [mesh/torus topology](#), a scalability of over 100,000 nodes, and [full-duplex](#) links that have a peak bandwidth of 10 GB/s (5 GB/s per direction)

Network on chip

The n by m frame based network on chip system

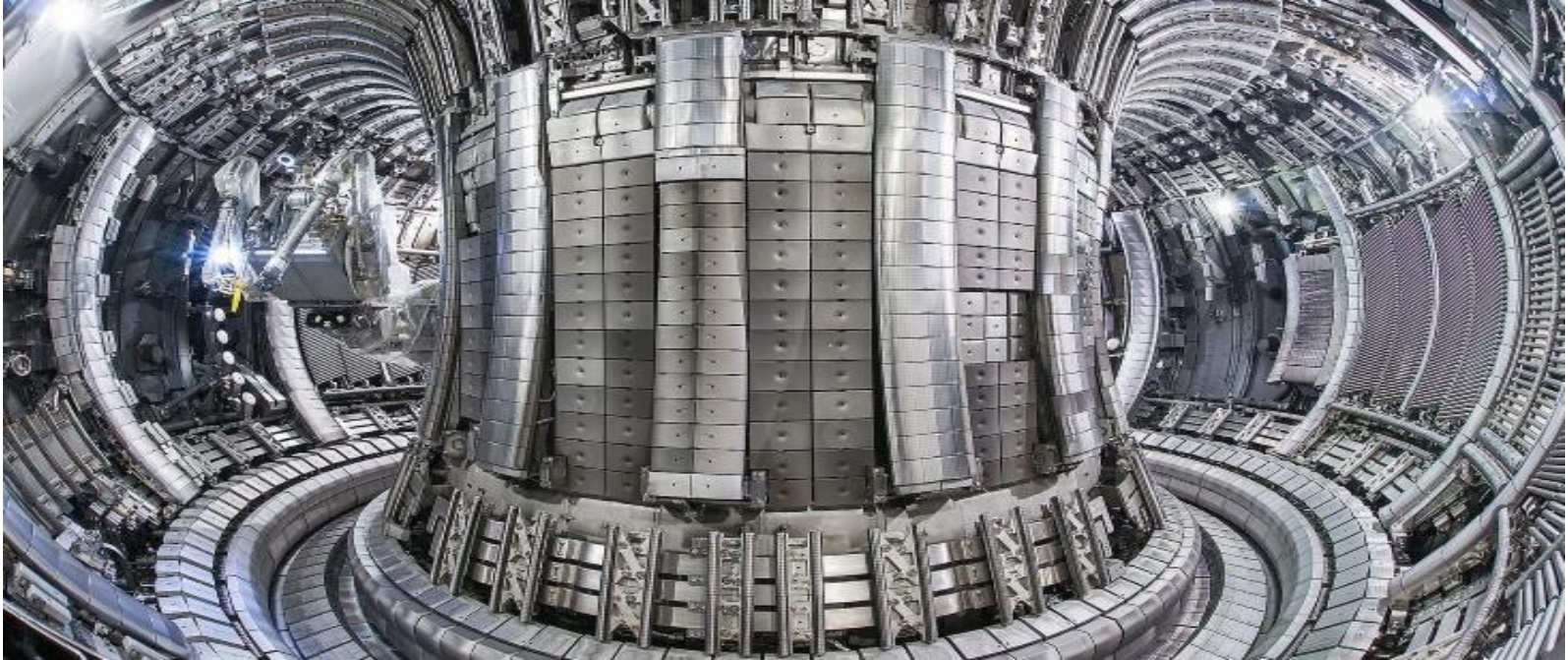


Method of finite elements



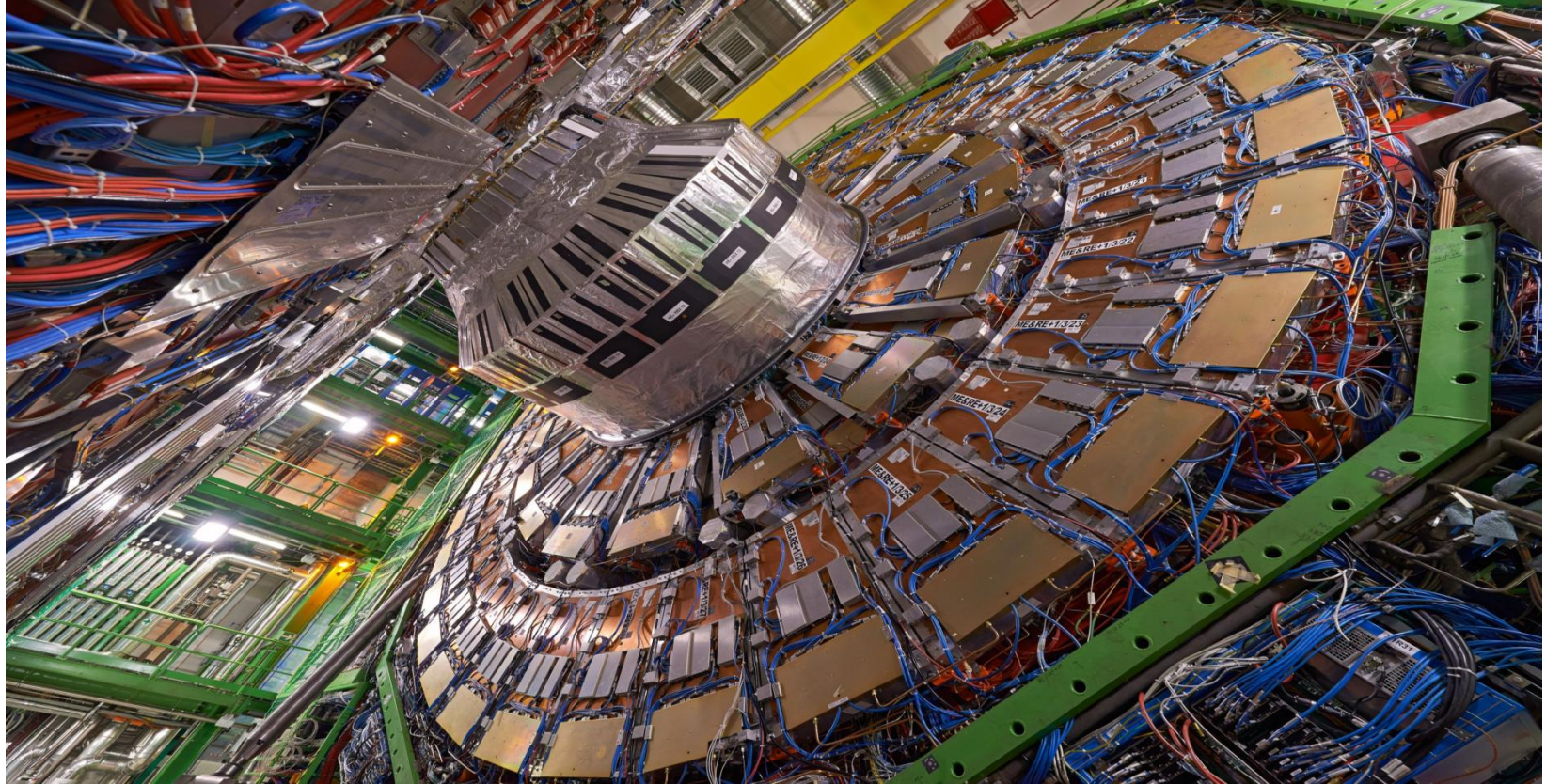
Each element on a core
or a grid node

Controlled thermonuclear reaction



Tokamak (toroidal chamber with magnetic coils)

Colliders



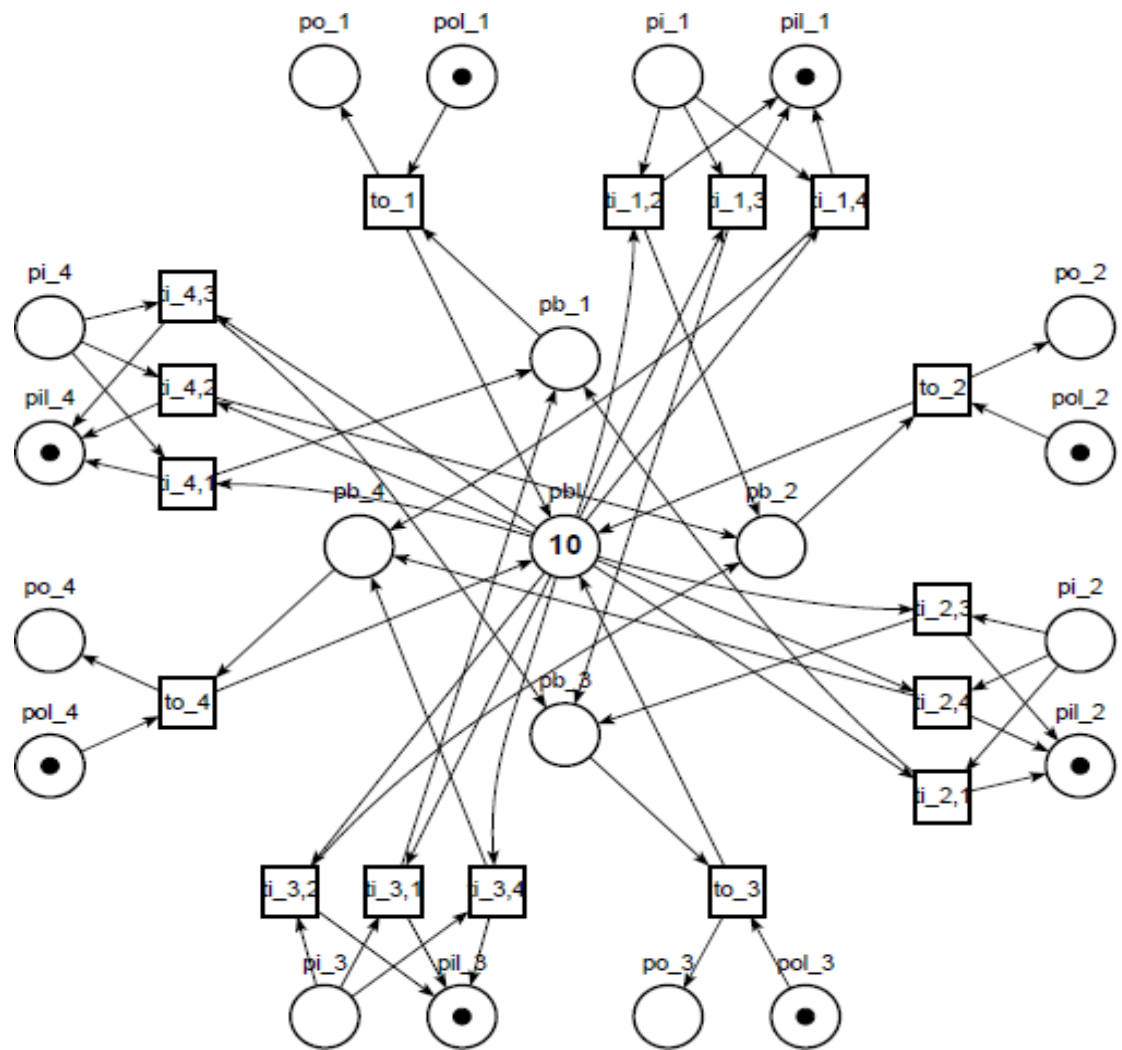
Studied structures

| Structure | System | Model | Analysis |
|-----------------------|---|-------|----------|
| Linear | Common bus Ethernet | 1987 | 2006 |
| Tree-like | Switched Ethernet | 2007 | 2007 |
| Rectangular | Computing Grid | 2008 | 2008 |
| Triangular, hexagonal | Radiobroadcasting, cellular networks | 2010 | 2010 |
| Hypercube | Computing Grid | 2008 | 2008 |
| Hypertorus | Computing Grid, Cloud | 2012 | 2012 |

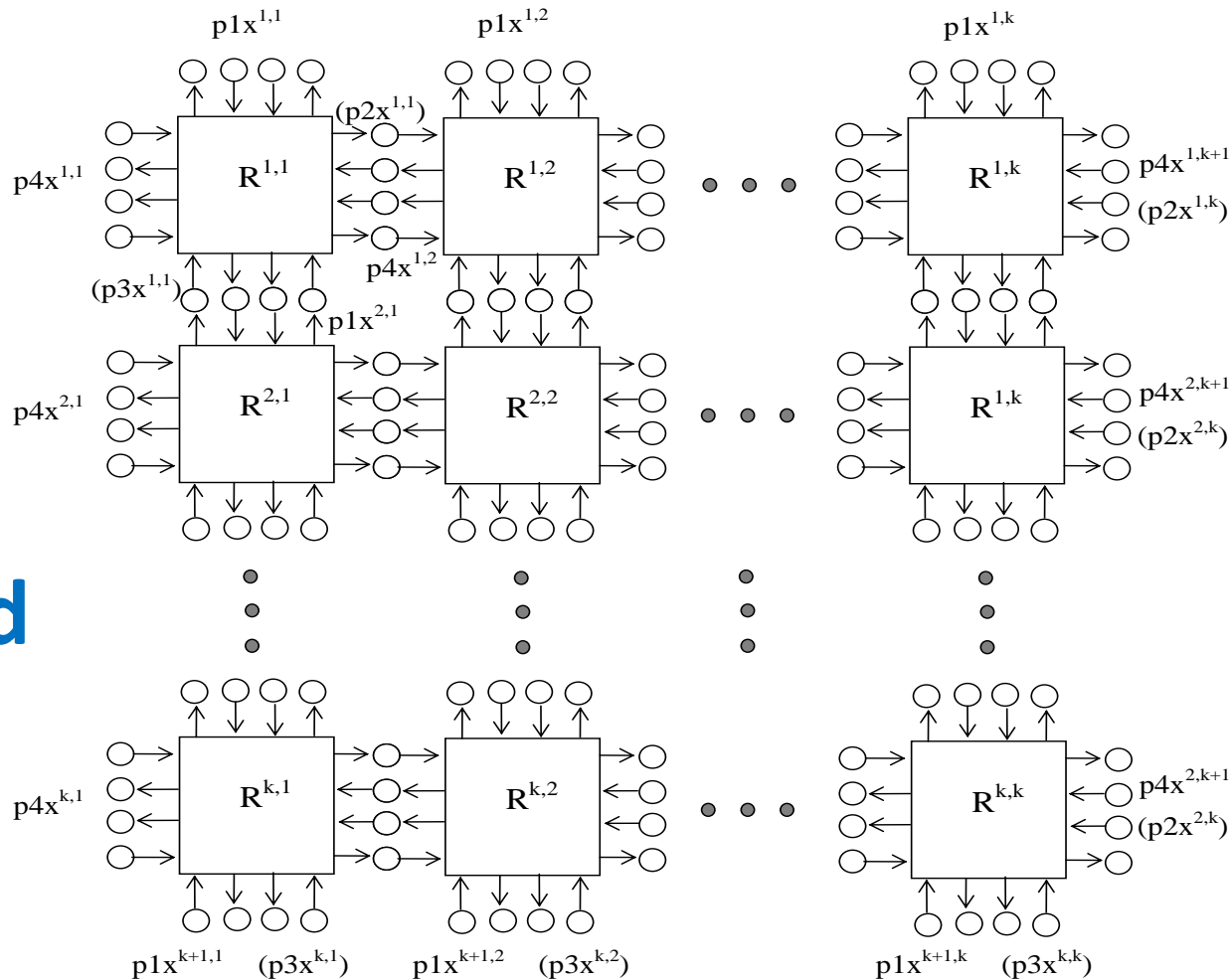
Protocol of grid switching node work

- **Simultaneous receiving and sending packets – duplex**
- **A node's port has buffers to store one receiving packet and one sending packet**
- **From the input port, a packet is situated for temporary storing into the internal buffer**
- **Capacity of the internal buffer is limited**
- **According to the destination address, packet is forwarded into the output port**
- **The output port is transmitting the packet over the communication channel**

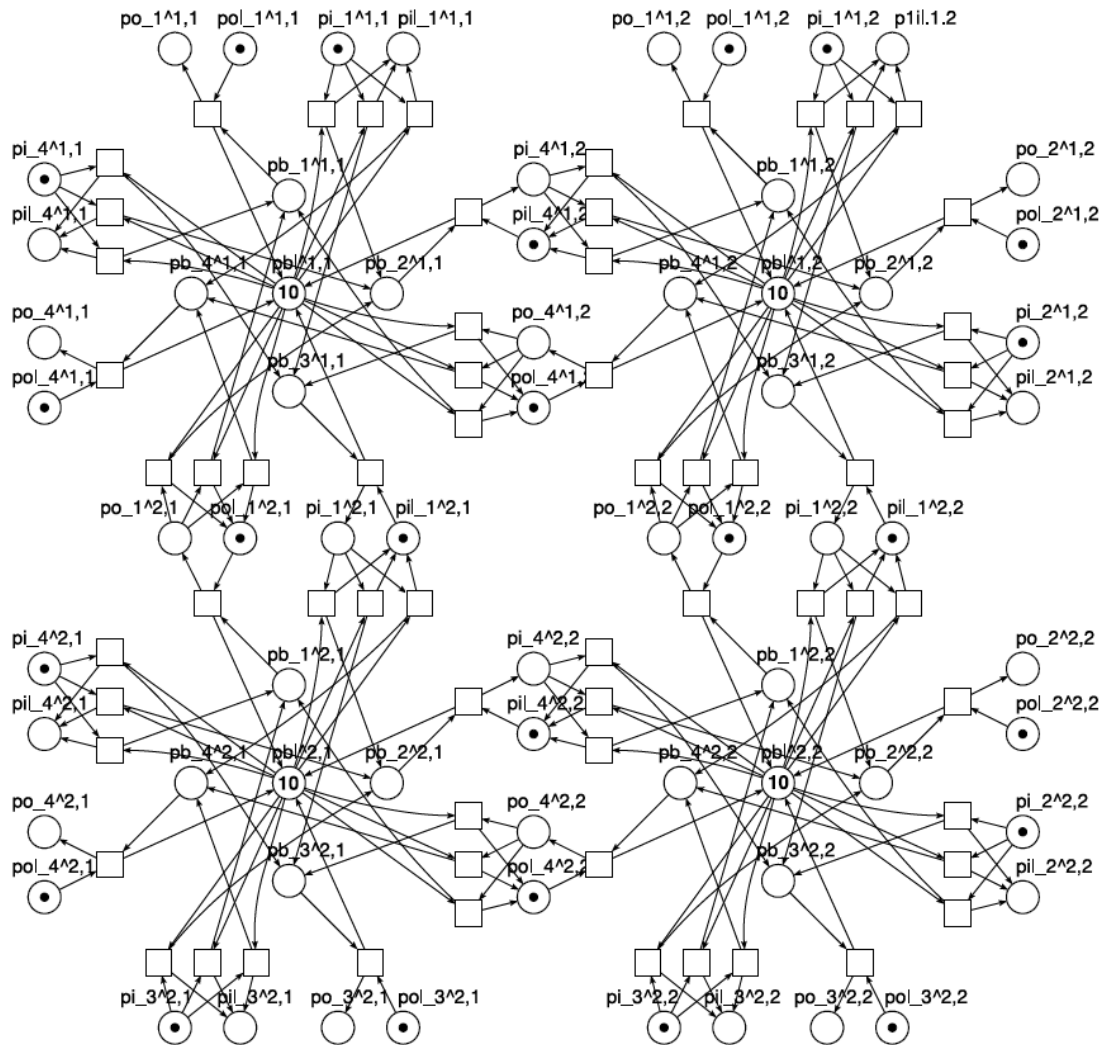
Model of square device of grid



Composition of rectangular grid



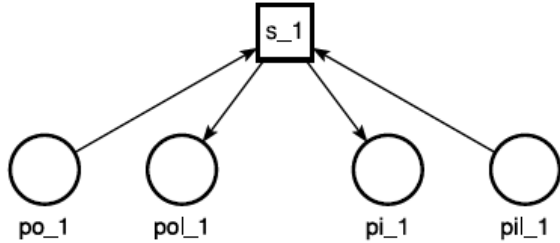
Открытая решётка 2 x 2



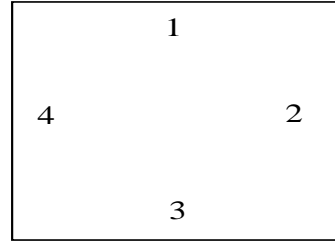
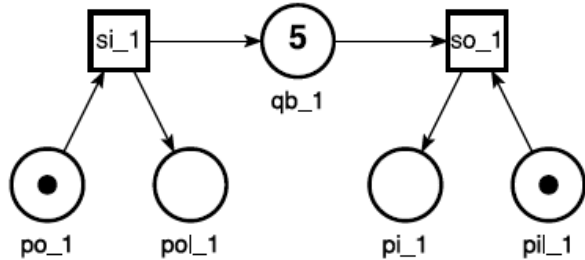
Border conditions

- Open grid
- Terminal (customer) device
- Connection of (opposite) borders
- Truncated communication devices on borders

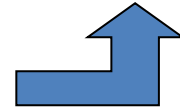
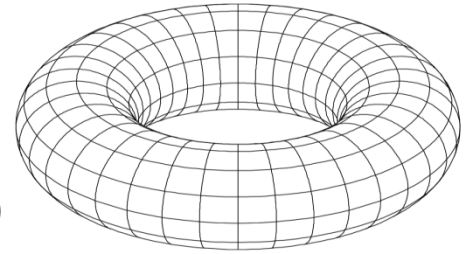
Examples of border conditions



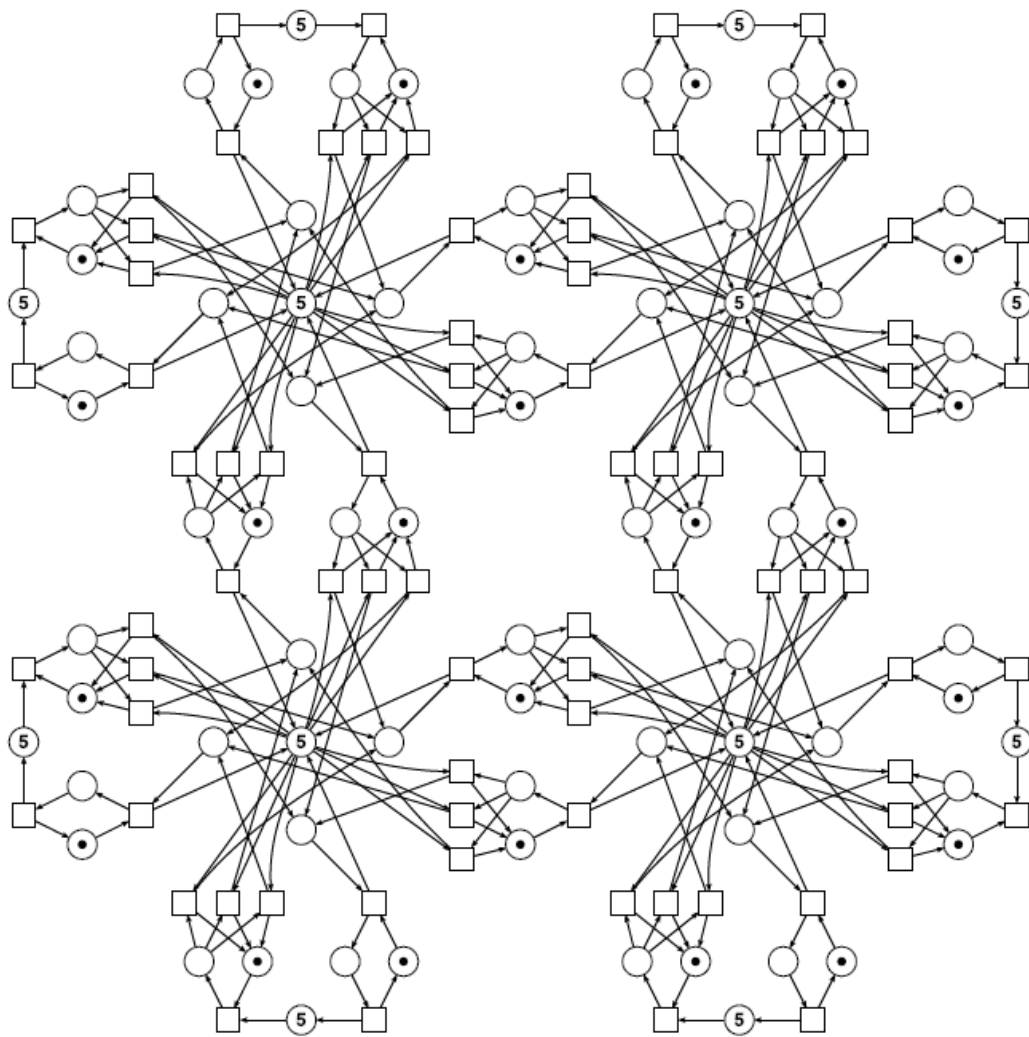
Terminal device



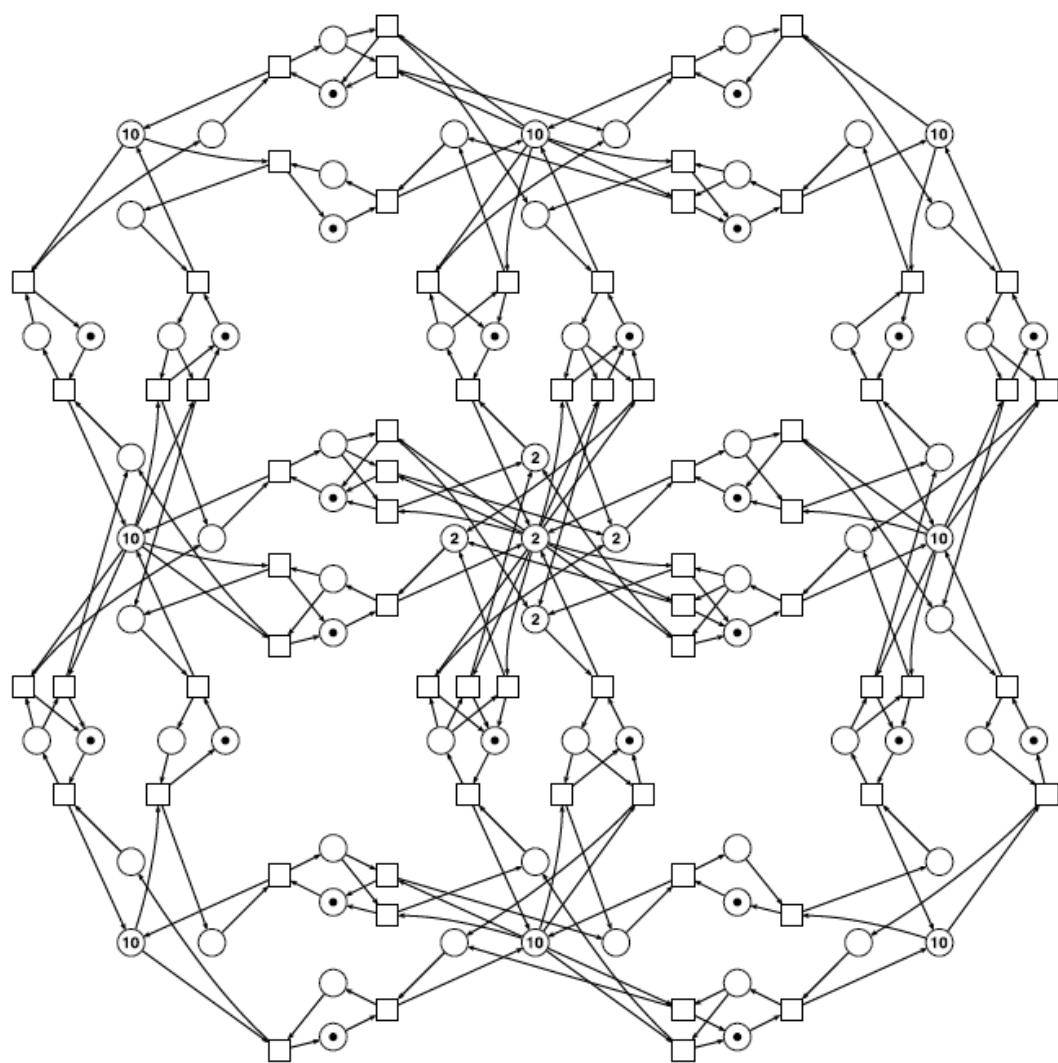
Connection of
borders (edges)



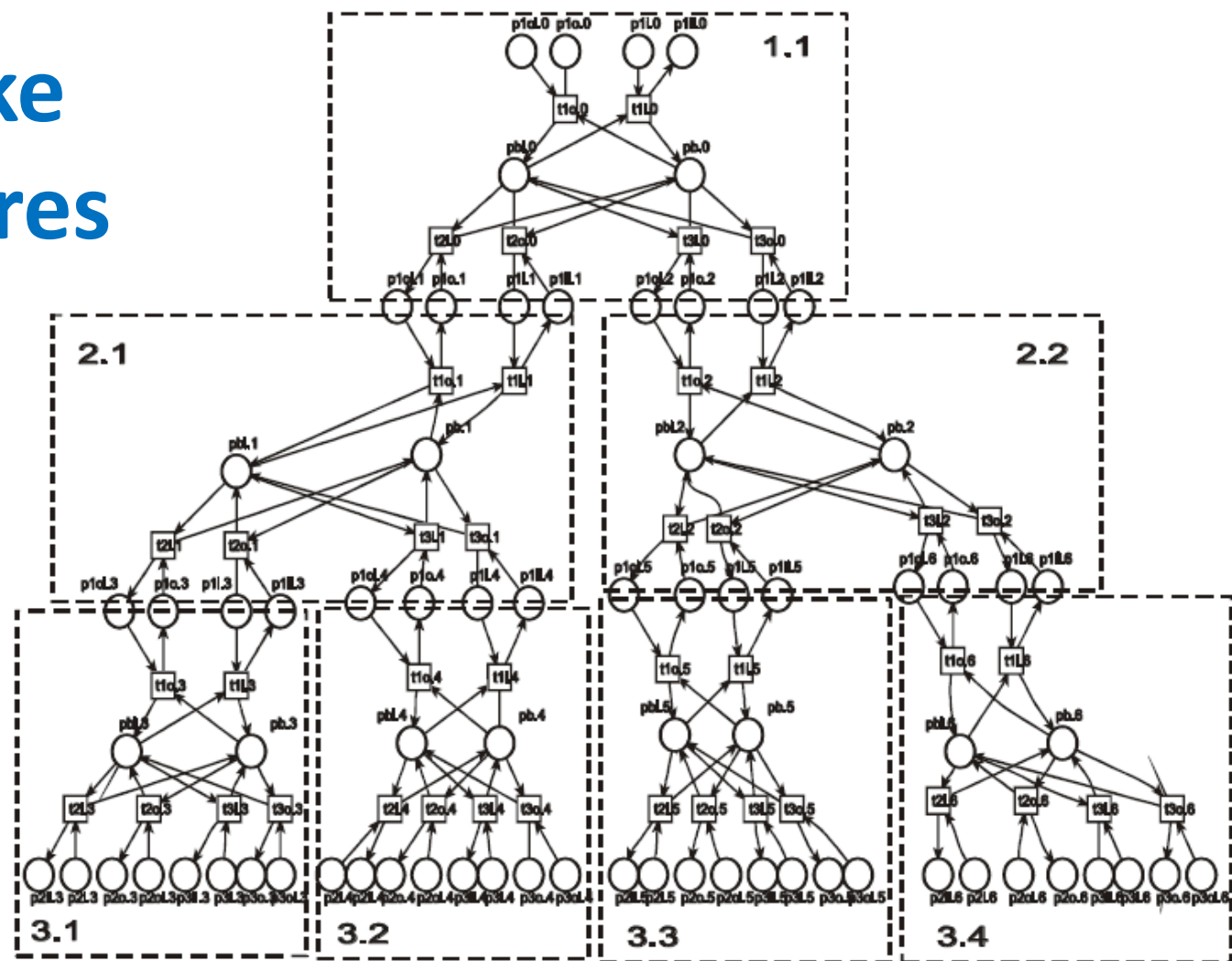
Closed grid
2 x 2



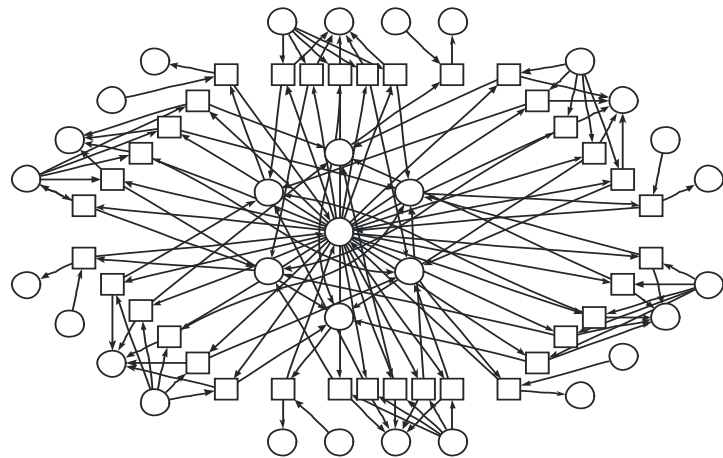
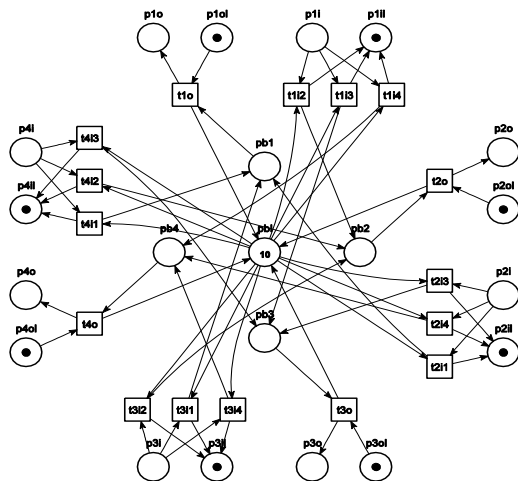
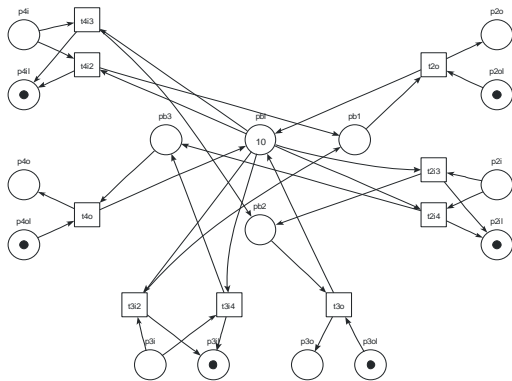
**Truncated
devices on
borders**



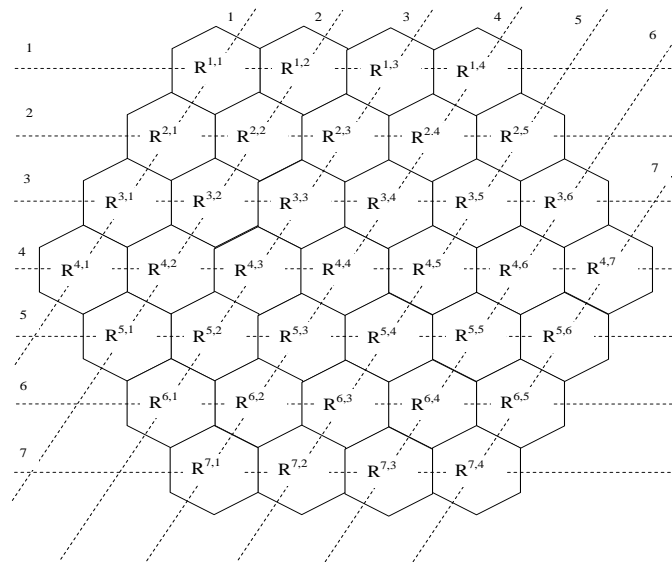
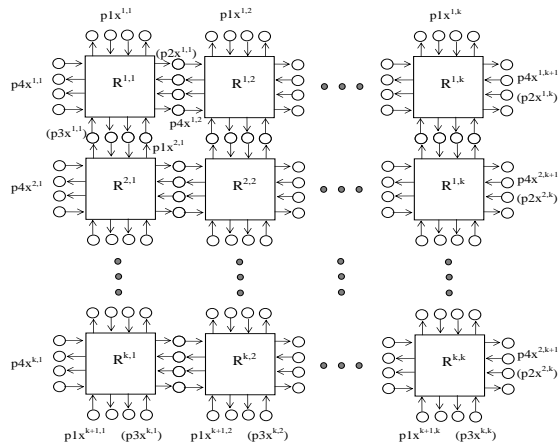
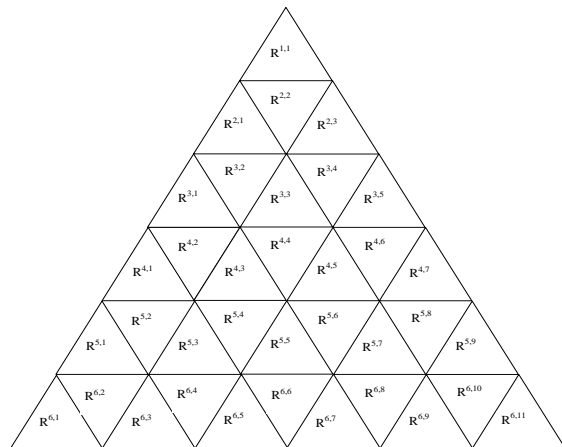
Treelike structures



Node shape: triangle, rectangle, hexagon



Composition of grids



Parametric specification of device (node)

$$\left(\left(\begin{array}{l} (to_u : pb_u, pol_u \rightarrow po_u, pbl) \\ (ti_{u,v} : pi_u, pbl \rightarrow pb_v, pil_u), \quad v = \overline{1, np}, \quad v \neq u \end{array} \right), \quad u = \overline{1, np} \right)$$

Parametric specification of open grid

$$\left(\begin{array}{l} (to_1^{i,j} : pol_1^{i,j}, pb_1^{i,j} \rightarrow po_1^{i,j}, pbl^{i,j}), \\ (ti_{1,v}^{i,j} : pi_1^{i,j}, pbl^{i,j} \rightarrow pil_1^{i,j}, pb_v^{i,j}), \\ v = 2, 3, 4, \\ (to_3^{i,j} : pil_1^{i+1,j}, pb_3^{i,j} \rightarrow pi_1^{i+1,j}, pbl^{i,j}), \\ (ti_{3,v}^{i,j} : po_1^{i+1,j}, pbl^{i,j} \rightarrow pol_1^{i+1,j}, pb_v^{i,j}), \\ v = 1, 2, 4, \\ (to_4^{i,j} : pol_4^{i,j}, pb_4^{i,j} \rightarrow po_4^{i,j}, pbl^{i,j}), \\ (ti_{4,v}^{i,j} : pi_4^{i,j}, pbl^{i,j} \rightarrow pil_4^{i,j}, pb_v^{i,j}), \\ v = 1, 2, 3, \\ (to_2^{i,j} : pil_4^{i,j+1}, pb_2^{i,j} \rightarrow pi_4^{i,j+1}, pbl^{i,j}), \\ (ti_{2,v}^{i,j} : po_4^{i,j+1}, pbl^{i,j} \rightarrow pol_4^{i,j+1}, pb_v^{i,j}), \\ v = 1, 3, 4 \end{array} \right), \quad i = \overline{1, k}, j = \overline{1, k}$$

**Infinite
system for
computing
place
invariants of
open grid**

$$\left\{ \begin{array}{l} to_1^{i,j} : -xpol_1^{i,j} - xpb_1^{i,j} + xpo_1^{i,j} + xpb^{i,j} = 0, \\ ti_{1,v}^{i,j} : -xpi_1^{i,j} - xpb^{i,j} + xpil_1^{i,j} + xpb_v^{i,j} = 0, v = 2, 3, 4, \\ to_3^{i,j} : -xpil_1^{i+1,j} - xpb_3^{i,j} + xpi_1^{i+1,j} + xpb^{i,j} = 0, \\ ti_{3,v}^{i,j} : -xpo_1^{i+1,j} - xpb^{i,j} + xpol_1^{i+1,j} + xpb_v^{i,j} = 0, \\ v = 1, 2, 4, \\ to_4^{i,j} : -xpol_4^{i,j} - xpb_4^{i,j} + xpo_4^{i,j} + xpb^{i,j} = 0, \\ ti_{4,v}^{i,j} : -xpi_4^{i,j} - xpb^{i,j} + xpil_4^{i,j} + xpb_v^{i,j} = 0, v = 1, 2, 3, \\ to_2^{i,j} : -xpil_4^{i,j+1} - xpb_2^{i,j} + xpi_4^{i,j+1} + pbl^{i,j} = 0, \\ ti_{2,v}^{i,j} : -xpo_4^{i,j+1} - xpb^{i,j} + xpol_4^{i,j+1} + xpb_v^{i,j} = 0, \\ v = 1, 3, 4; i = \overline{1, k}, j = \overline{1, k}. \end{array} \right.$$

Place invariants in parametric form

$$\left(\begin{array}{l}
 (pi_1^{ij}, pil_1^{ij}), i = \overline{1, k+1}, j = \overline{1, k}; \\
 (po_1^{ij}, pol_1^{ij}), i = \overline{1, k+1}, j = \overline{1, k}; \\
 (pi_4^{ij}, pil_4^{ij}), i = \overline{1, k}, j = \overline{1, k+1}; \\
 (po_4^{ij}, pol_4^{ij}), i = \overline{1, k}, j = \overline{1, k+1}; \\
 (pb_1^{ij}, pb_2^{ij}, pb_3^{ij}, pb_4^{ij}, pbl^{ij}), i = \overline{1, k}, j = \overline{1, k}; \\
 (((pil_1^{ij}, pol_1^{ij}, pil_4^{ij}, pol_4^{ij}, pbl^{ij}), i = \overline{1, k}, j = \overline{1, k}); \\
 ((pil_4^{k+1}, pol_4^{k+1}), i = \overline{1, k}), ((pil_1^{k+1,j}, pol_1^{k+1,j}), j = \overline{1, k})) \\
 (((pi_1^{ij}, po_1^{ij}, pi_4^{ij}, po_4^{ij}, ((pb_u^{ij}), u = \overline{1, 4})), \\
 \quad i = \overline{1, k}, j = \overline{1, k}); \\
 ((pi_4^{k+1}, po_4^{k+1}), i = \overline{1, k}), ((pi_1^{k+1,j}, po_1^{k+1,j}), j = \overline{1, k}))
 \end{array} \right).$$

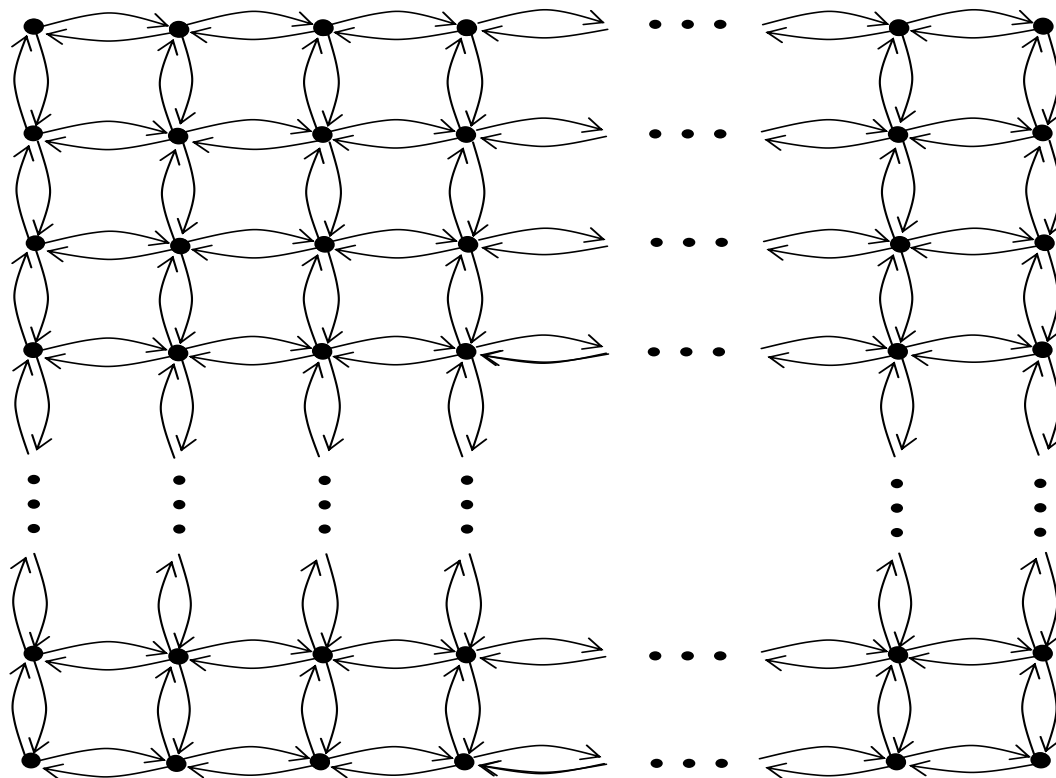
Basic results

- **Invariance of hypercube and hypertorus has been proven in multidimensional grid of any size and any number of dimensions**
- **Invariance of triangular, hexagonal, and treelike grids of any size has been proven**
- **Place invariant grid is structurally conservative and bounded**
- **Transition invariant grid is structurally stationary repetitive**

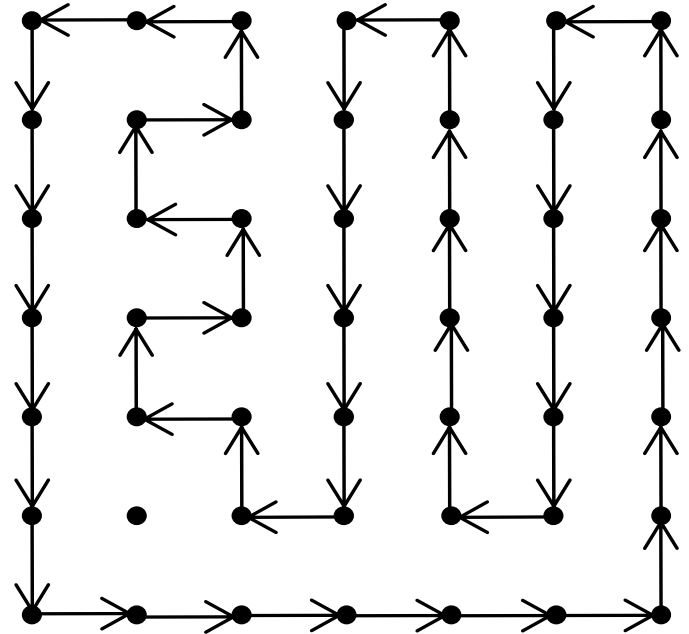
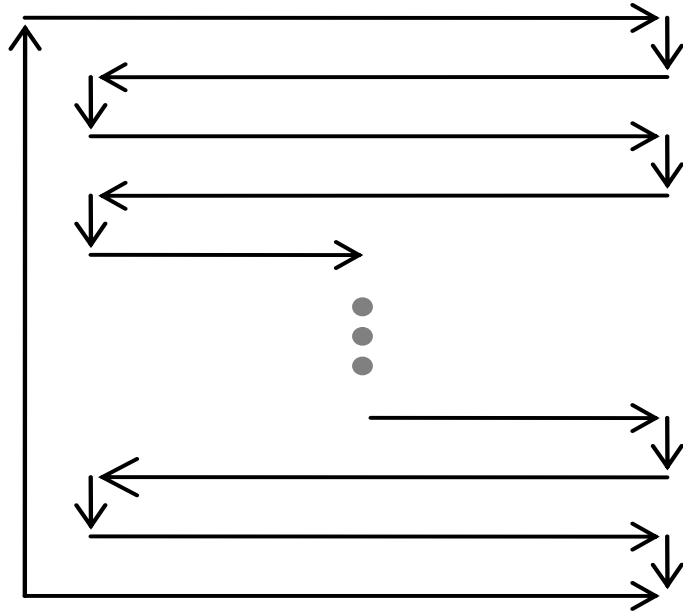
Methods of grid analysis

- **Solve infinite linear Diophantine systems of equations and inequalities to place/transition invariants**
- **Construct explicitly cyclic transition firing sequences**
- **Domain specific: auxiliary graphs of directions of packet transmission and possible blockings of devices**

Graph of possible blockings



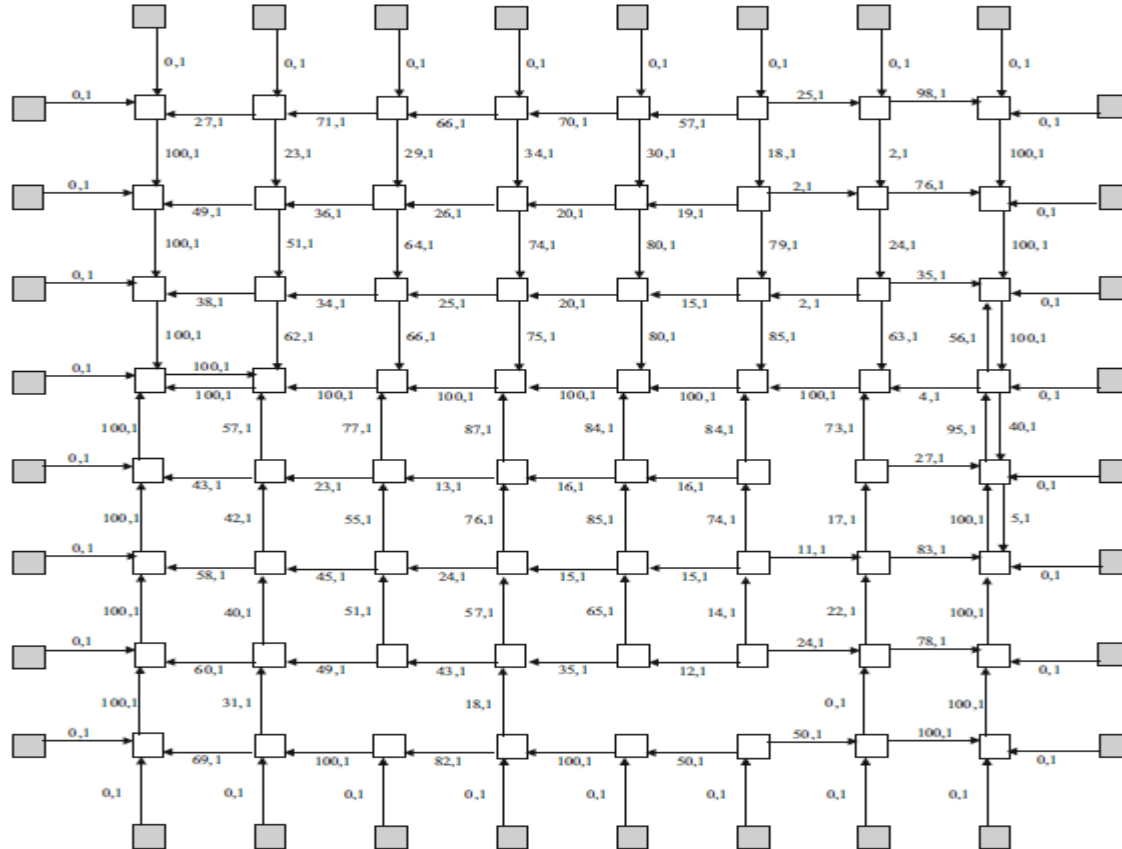
Detours of the possible blockings graph



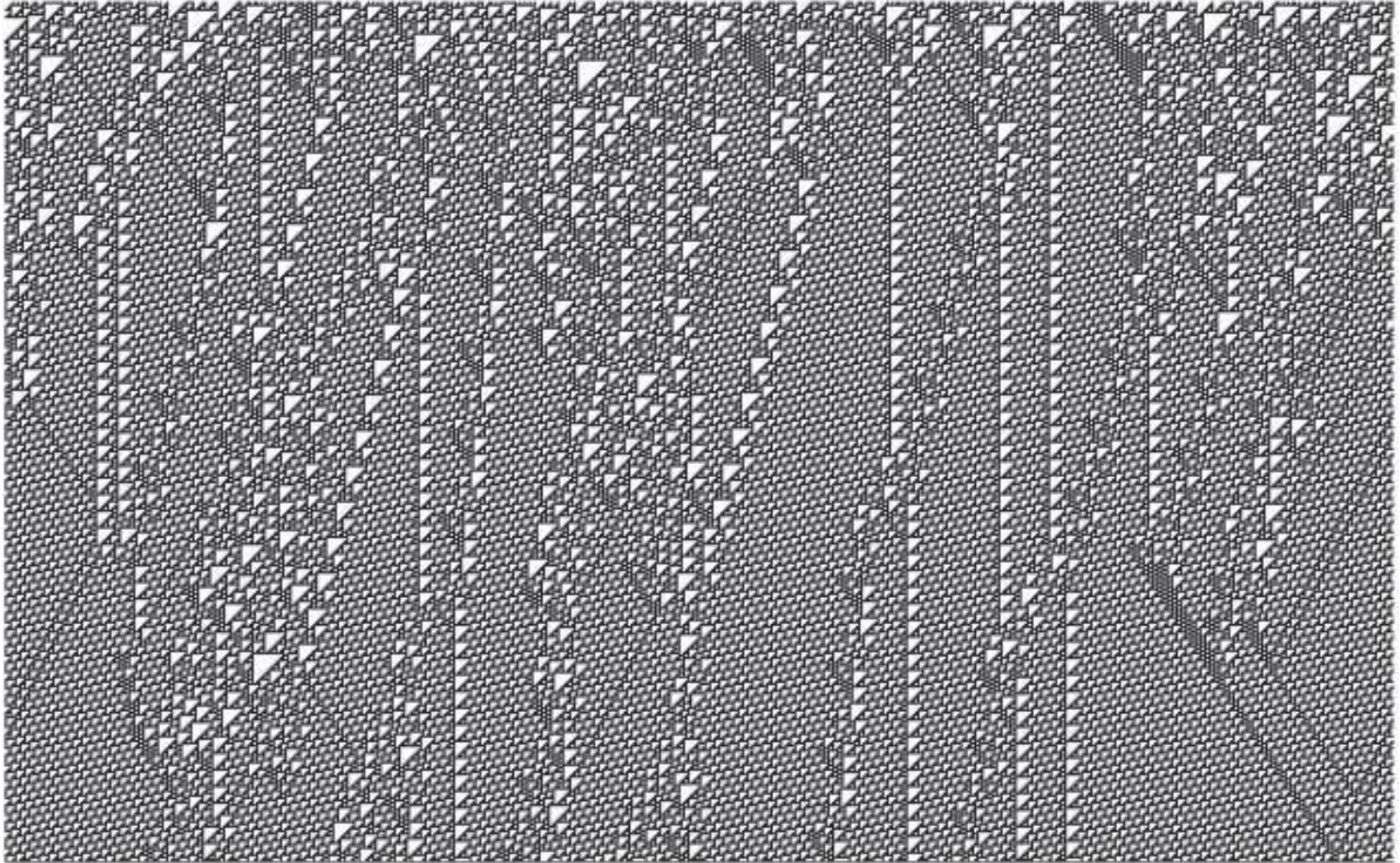
Compound deadlocks reveled - cybersecurity

- **Cycle of blockings**
- **Isolation of vertex**
- **Chain of blockings that ends on an early blocked chain (vertex)**
- **Avalanche-like character of the blocked vertices number**
- **Possibility of blocking a network (grid) by ill-intentioned traffic**

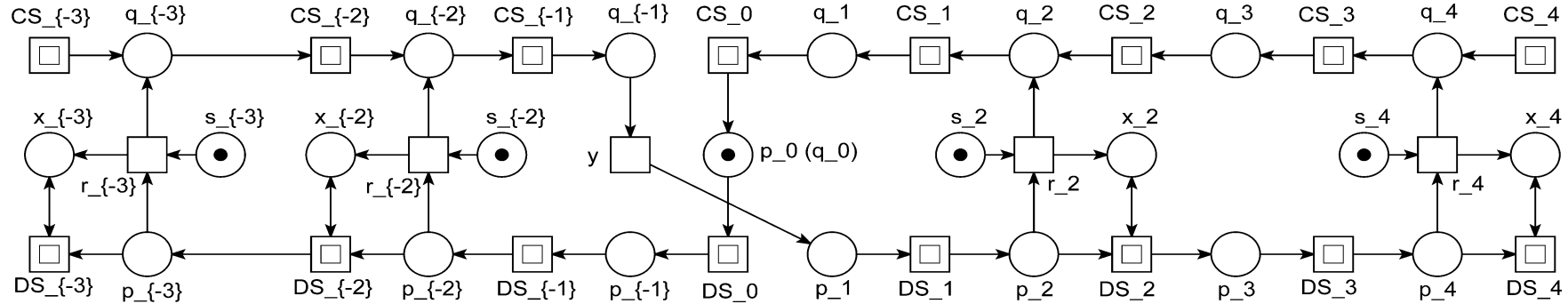
Deadlock example



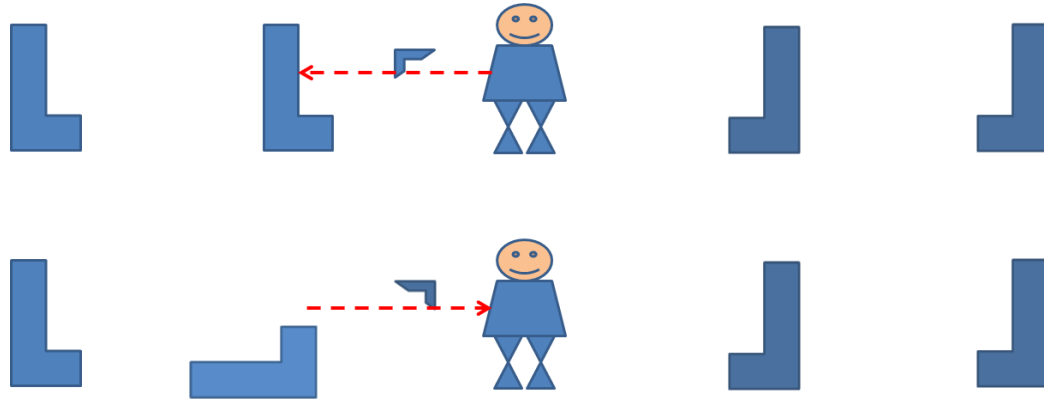
Modeling of cellular automata



Modeling of cellular automata 110



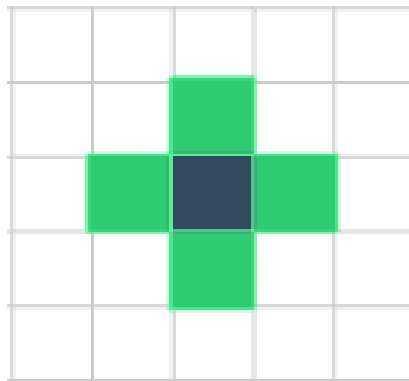
**Boomerangs
and
barriers**



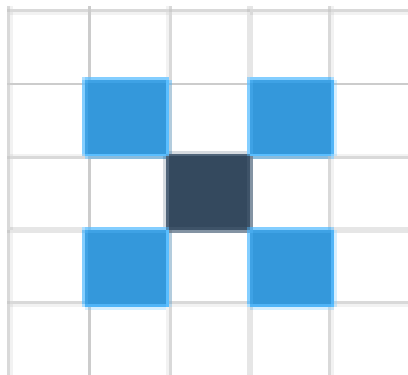
Generalized neighborhood for cellular automata and grids

- **Von Neumann neighborhood – Manhattan distance (sum of differences absolute values on coordinates)**
- **Moore neighborhood – Chebyshev distance (maximum on absolute values of differences on coordinates)**
- **Generalized neighborhood – Chebyshev distance 1 combined with – Manhattan distance 1 not greater than k**

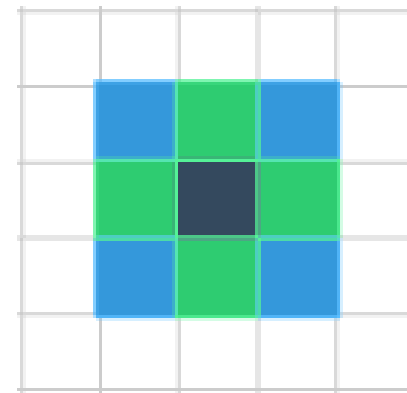
Two dimensions (2D)



1-neighborhood,
sharp 1-neighborhood,
Von Neumann neighborhood,
4 neighbors

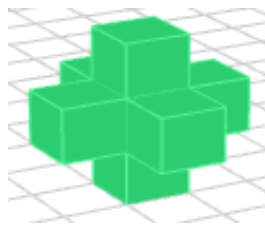


sharp 2-neighborhood
4 neighbors

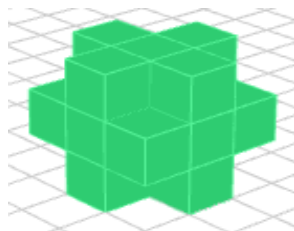


2-neighborhood,
Moore neighborhood,
8 neighbors

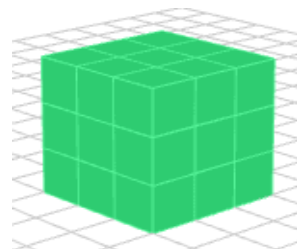
Three dimensions (3D)



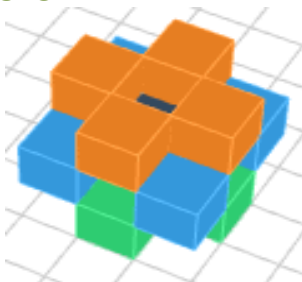
1-neighborhood,
sharp 1-neighborhood,
Von Neumann neighborhood,
6 neighbors



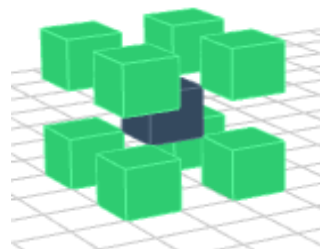
2-neighborhood,
18 neighbors



3-neighborhood,
Moore neighborhood,
26 neighbors



sharp 2-neighborhood,
12 neighbors



sharp 3-neighborhood,
8 neighbors

Problem

- **Efficient systems and networks design**
- **Express-analysis of performance and quality of service of networks, grids, and clouds in the process of model-driven design**

References

<http://daze.ho.ua>

- Dmitry A. Zaitsev, Tatiana R. Shmeleva & Birgit Proll, Spatial specification of hypertorus interconnect by infinite and reenterable coloured Petri nets, [International Journal of Parallel, Emergent and Distributed Systems](#), 37(1), 2022, 1-21.
- Zaitsev, Dmitry A., Tatiana R. Shmeleva, and David E. Probert. 2021. "Applying Infinite Petri Nets to the Cybersecurity of Intelligent Networks, Grids and Clouds" [Applied Sciences](#) 11, no. 24: 11870.
- D.A. Zaitsev, I.D. Zaitsev, and T.R. Shmeleva, [Infinite Petri Nets: Part 2, Modeling Triangular, Hexagonal, Hypercube and Hypertorus Structures](#), [Complex Systems](#), 26(4), 2017, 341-371.
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