

Vistula, IT Faculty, 2014

Algorithms and Complexity

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Lecture 11:

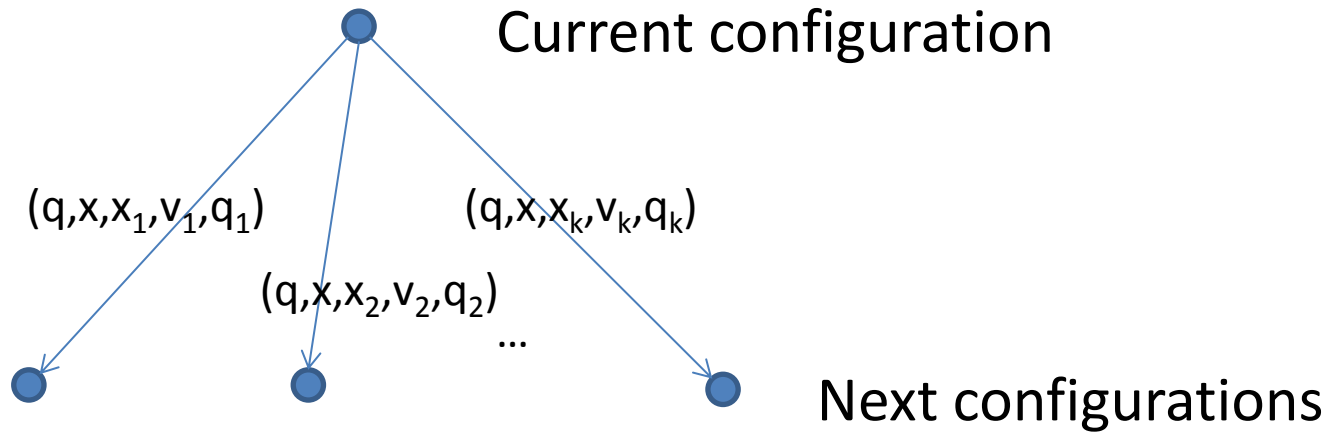
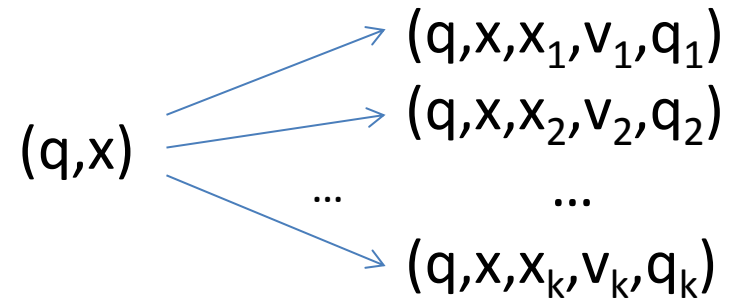
Nondeterministic Turing machine

Nondeterministic transition function

More than one instruction for the same combination (q,x)

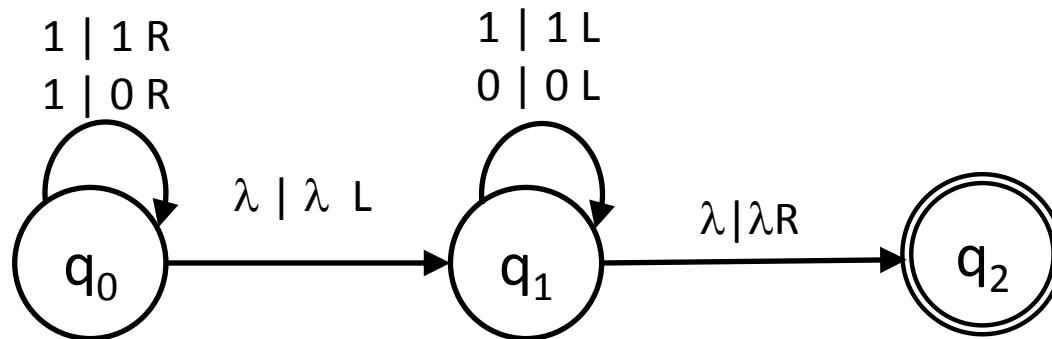
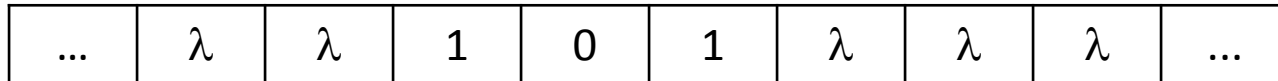
Transition relation

$$T \subseteq (Q \times X) \times (X \times V \times Q)$$



An example of NDTM

Generate an arbitrary binary number of specified length



Solving a mass algorithmic problem with NDTM

- The answer of NDTM is positive if there is at least one sequence of valid configurations that halts in a final state with a positive answer.
- Computation of NDTM could be considered as a branching process – a tree of valid configurations. If a branch accepts an input word w than we say that NDTM accepts the word w

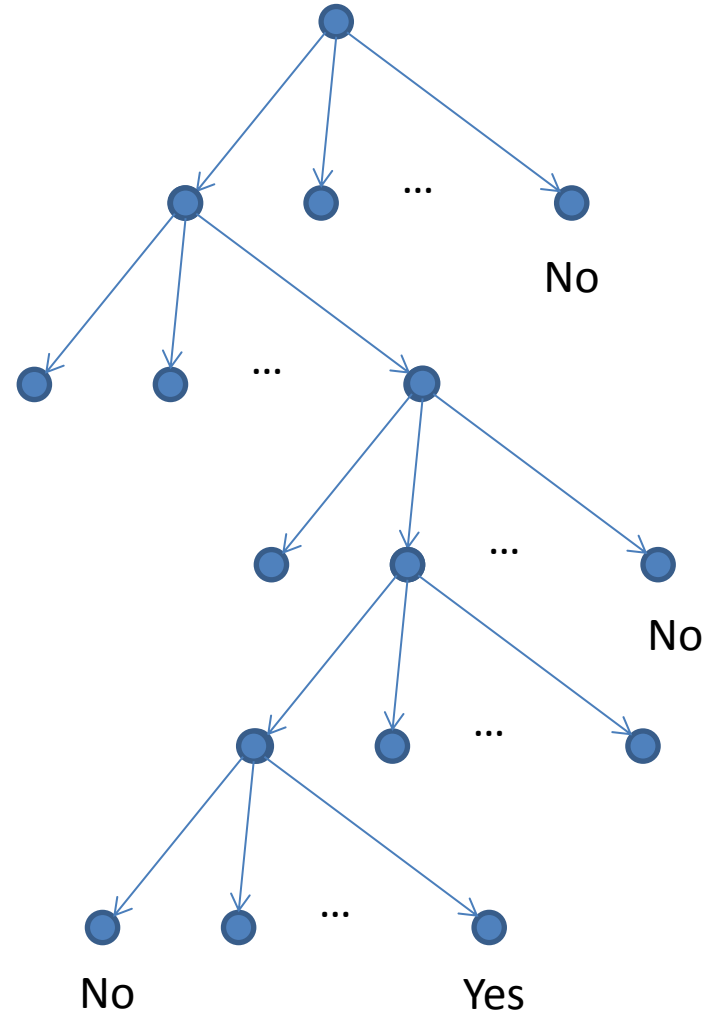
Comparing TM and NDTM computations

TM



Yes or No

NDTM – result Yes



P and NP problems

Time complexity of NDTM – the minimal number of steps leading to a positive answer or the maximal number of steps leading to all the negative answers.

Class P – is accepted by TM in a polynomial time

Class NP – is accepted by NDTM in a polynomial time

Simulating NDTM with TM

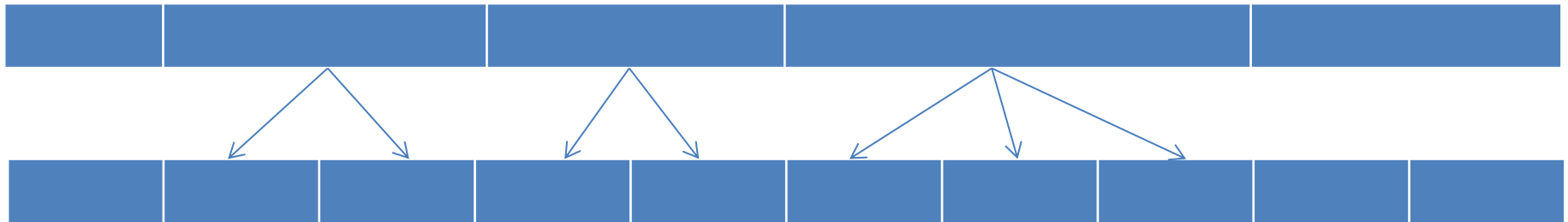
A NDTM with time complexity t is simulated with TM with time complexity $O(c^t)$

The proof idea. Simulate computations of NDTM with tree tape TM enumerating configurations of the current and the next step on the first two tapes and copying the next into the current after a step completion. The third tape is working.

A configuration $s = \alpha q x \beta$

A tape: s_1, s_2, \dots, s_n

1. Whether tape1 contains a final configuration?
2. Construct tape2 on tape1



3. Copy tape2 into tape1

Simulating NDTM with TM – number of configurations

Suppose that the number of instructions for each pair (q,x) is bounded with c then after the first step we have no more than c next configurations, after the second step no more than c^2 configurations and after step t no more than c^t final configurations.

The total number of vertices in the tree does not exceed 2^c .
Finally we have $O(2^t)$.