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Algorithms and Complexity

Dmitry A. Zaitsev

<http://daze.ho.ua>

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A first pass - roughly

Introduction to algorithms and complexity

- Definition of algorithm
- Features of algorithm
- Hilbert list and necessity of formalization
- Known models of computations
- Correctness and complexity
- Undecidable, intractable, and tractable
- Algorithms and programs

A case study: paths in a triangle of numbers

- A task – maximal sum on a path in a triangle
- Exhaustive search algorithm
- Programming and testing
- Storing intermediate sums algorithm
- Programming and testing
- Generating source data
- Running programs to estimate time

Turing machine as a model of computations

- Description of TM: tape and control head
- Rules of TM work
- Examples of TM
- TM representation: graph, table, instructions
- Thesis of Church
- TM encoding
- Universal TM as a prototype of computers

Undecidable problems

- Self-applicability of TM
- Proof that SATM problem is undecidable
- Reduction technique to prove undecidability
- TM halting problem
- List of undecidable problems
- How to do with undecidable problems
- Other models of computation

Time and space complexity of algorithms

- Time and space complexity of TM
- Time and space complexity of a program
- Best, worse, and average complexity
- Asymptotic estimations of complexity
- Linear and logarithmic scale
- Calculating time complexity
- Calculating space complexity

Arrays, tables, and lists

- One dimension array and matrix
- D-dimension arrays
- Tables and search
- One link lists
- Two link lists
- Basic operations
- Stacks and queues

Graphs and trees

- Concept of a graph
- Adjacency and incidence matrices, lists
- Examples of graph algorithms
- An exhaustive search tree
- Depth and width detours
- Binary tree to store data
- Numerical trees

Recursive algorithms

- Factorial and Fibonacci numbers
- Hanoi tower puzzle
- Tracing recursive calls
- Complex and mutual recursion
- Recursive algorithms for lists
- Recursive algorithms for trees

Sorting, merging, and string algorithms

- Sorting via sequential minimums
- Estimation of complexity
- Merging two arrays
- Fast sorting
- Other sorting algorithms
- Substring matching

Search algorithms

- Sequential search
- Dichotomy
- Binary tree search
- Hash tables
- Numerical tree search

Nondeterministic Turing machine

- Definition of NDTM
- Examples of NDTM
- P and NP problems
- Converting NDTM to TM
- Nondeterministic choice in algorithms

NP-complete problems

- Whether $P=NP$?
- Other complexity classes
- NP-completeness
- Cook theorem
- List of NP-complete problems
- How to do with NP-complete problems

Divide and conquer

- Divide-and-conquer principle
- Multiplication of integers
- Sorting and merging arrays
- Matrix multiplication
- Branch and bound approach – to limit the number of variants
- Bellman-Ford algorithm
- Travelling salesman problem

Dynamic programming

- Principles of DP: store and reuse intermediate results
- Simplest example – calculating Fibonacci numbers
- Paths in a triangle of numbers
- Paths in directed acyclic graphs
- Knapsack problems
- Chain matrix multiplication

Paradigms of programming

- Imperative and declarative approaches
- Structural programming
- Object-oriented programming
- Functional programming
- Logical programming
- Data flow programming
- Parallel and distributed programming

Literature

- Thomas H. Cormen, Charles E. Leiserson, Ronald L. Rivest and Clifford Stein, Introduction to Algorithms, The MIT Press
- Alfred V. Aho, John E. Hopcroft, Jeffrey D. Ullman, The Design and Analysis of Computer Algorithms, Addison-Wesley
- László Lovász, Complexity of Algorithms, Eötvös Loránd University
- Herbert S. Wilf, Algorithms and Complexity, A K Peters/CRC Press
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A second pass – thoroughly
by lectures